

Parallel Algorithms

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WARNING!

- this talk is **not** about how to **implemented** parallel algorithms but about how to **use** them
- this talk is about C++

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Lots of Concurrency

- number of cores keeps growing
- different concurrency approaches are available
 - GPU concurrency for parallelism
 - FPGAs for specialised operations

C++ is Sequential

- statements are executed in sequence
- even when operations are independent:
 - hard for compilers to detect non-trivial cases
 - order may be required accidentally
- => need to express asynchronicity potential

Example

```
for (auto it = begin; it != end; ++it, ++to) {  
    *to = fun(*it);  
}
```

- can be parallel if `fun()` doesn't have side-effects
- `size` is reasonably large or `fun()` takes long

Use OpenMP

```
#pragma omp parallel for  
for (auto it(begin); it < end; ++it, ++to) {  
    *to = fun(*it);  
}
```

- outside the language and doesn't quite fit
- it is unspecified if parallel executions nest
- only works with random access iterators

Use `std::thread`

```
std::vector<std::thread> ts;
for (auto it(begin), e(begin); it != end; it = e) {
    e += std::min(std::distance(it, end), buf);
    ts.emplace_back( [= ]() {
        for (; it != e; ++it, ++to) {
            *to = fun(*it);
        }
    });
}
for (auto& t: ts) { t.join(); }
```

- not easy to use

Use `std::async`

```
std::vector<std::future<void>> fs;
for (auto it(begin), e(begin); it != end; it = e) {
    e += std::min(std::distance(it, end), buf);
    fs.emplace_back(std::async( [= ]() {
        for (; it != e; ++it, ++to) {
            *to = fun(*it);
        }
    }));
}
for (auto& f: fs) { f.get(); }
```

- not easy to use

Use TBB

```
using range = tbb::blocked_range<int>;  
tbb::parallel_for(range(0, end - begin),  
    [=](range const& r){  
        for (auto i(r.begin()); i != r.end(); ++i) {  
            to[i] = fun(begin[i]);  
        });  
});
```

- some algorithms are easier to use
- a reasonable direction

Parallel Algorithm

```
std::transform(std::par, begin, end, to, fun);
```

- can use different policies (std::seq, std::par, ...)
- assumptions about the parameters are made:
 - parameter calls don't introduce data races
 - parameters can be copied (not just moved)

Status Quo

- algorithms execute sequentially

```
std::for_each(begin, end, fun);
```

```
std::transform(begin, end, to, fun);
```

```
std::inclusive_scan(begin, end, to, op);
```

Objective

- enable easy parallel execution

```
std::for_each(policy, begin, end, fun);
```

```
std::transform(policy, begin, end, to, fun);
```

```
std::inclusive_scan(policy, begin, end, to, op);
```

Possibly Not That Easy

1. nobody uses algorithms
2. potential of improvements depends on use
 - no point parallelising fast executing small loops
3. parallel execution may introduce data races
 - through iterators or function objects

Concurrency Model

- pass *execution policy* to allow concurrency
 - type indicates permitted approaches
- *element access functions* obey policy-specific constraints
- implementation *may* take advantage of these
 - ... but is *not* required to do so

Element Access Functions

functions used on parameters:

- any iterator operation according to its category
- operations specified to be used on elements
- specified uses of function objects
- required operations on function objects

Execution Policy

- `std::is_execution_policy<T>::value` for detection
- `std::sequenced_policy` `std::seq`
- `std::parallel_policy` `std::par`
- `std::parallel_unsequenced_policy` `std::par_unseq`

std::seq

- sequential execution
- primarily intended for debugging
- same common constraints and interface changes
 - exceptions result in `std::terminate()`
 - no [required] support for input iterators
 - changed return types

std::par

- allow parallel [threaded] execution
- element access functions shall not introduce data races
 - they *can* use locks (when really necessary)
 - no interleaved execution

std::par_unseq

- allow parallel, interleaved execution
 - for example using multiple threads on a GPU
- element access functions shall not introduce data races and have no order dependency
 - they *cannot* use locks

Supported Algorithms

all algorithms for which concurrent execution may be a benefit are supported

- no support for sub-linear algorithms
- some algorithms use different names
- some algorithms are rarely used and complicated to parallelise
- some oddballs are not supported

Algorithms

accumulate
adjacent_difference
adjacent_find
all_of
any_of
binary_search
clamp
copy
copy_backward
copy_if
copy_n
count
count_if
destroy
destroy_at
destroy_n
equal
equal_range
exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n
gcd
generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned
is_permutation
is_sorted
is_sorted_until
iter_swap
lcm
lexicographical_compare
lower_bound
make_heap
max
max_element
merge
min
min_element
minmax
minmax_element
mismatch
move
move_backward
next_permutation
none_of
nth_element

partial_sort
partial_sort_copy
partial_sum
partition
partition_copy
partition_point
pop_heap
prev_permutation
push_heap
reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy
sample
search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union
shuffle

sort
sort_heap
stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy
upper_bound

Algorithms: $O(1)$

accumulate
adjacent_difference
adjacent_find
all_of
any_of
binary_search

clamp

copy
copy_backward

copy_if

copy_n

count

count_if

destroy

destroy_at

destroy_n

equal

equal_range

exclusive_scan

fill

fill_n

find

find_end

find_first_of

find_if

find_if_not

for_each

for_each_n

gcd

generate

generate_n

includes

inclusive_scan

inner_product

inplace_merge

iota

is_heap

is_heap_until

is_partitioned

is_permutation

is_sorted

is_sorted_until

iter_swap

lcm

lexicographical_compare

lower_bound

make_heap

max

max_element

merge

min

min_element

minmax

minmax_element

mismatch

move

move_backward

next_permutation

none_of

nth_element

partial_sort

partial_sort_copy

partial_sum

partition

partition_copy

partition_point

pop_heap

prev_permutation

push_heap

reduce

remove

remove_copy

remove_copy_if

remove_if

replace

replace_copy

replace_copy_if

replace_if

reverse

reverse_copy

rotate

rotate_copy

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search

search_n

set_difference

set_intersection

set_symmetric_difference

set_union

shuffle

sort

sort_heap

stable_partition

stable_sort

swap_ranges

transform

transform_exclusive_scan

transform_inclusive_scan

transform_reduce

uninitialized_copy

uninitialized_copy_n

uninitialized_default_construct

uninitialized_default_construct_n

uninitialized_fill

uninitialized_fill_n

uninitialized_move

uninitialized_move_n

uninitialized_value_construct

uninitialized_value_construct_n

unique

unique_copy

upper_bound

Algorithms: $O(\ln n)$

accumulate
adjacent_difference
adjacent_find
all_of
any_of

binary_search

copy
copy_backward
copy_if
copy_n
count
count_if
destroy

destroy_n
equal
equal_range

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned
is_permutation
is_sorted
is_sorted_until

lexicographical_compare
lower_bound

make_heap
max_element
merge

min_element

minmax_element
mismatch
move
move_backward
next_permutation
none_of
nth_element

partial_sort
partial_sort_copy
partial_sum

partition
partition_copy
partition_point
pop_heap
prev_permutation
push_heap

reduce
remove
remove_copy
remove_copy_if
remove_if

replace
replace_copy
replace_copy_if
replace_if

reverse
reverse_copy
rotate
rotate_copy

sample
search
search_n
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set_union
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sort
sort_heap
stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy
upper_bound

Algorithms: heap

accumulate
adjacent_difference
adjacent_find
all_of
any_of

copy
copy_backward
copy_if
copy_n
count
count_if
destroy

destroy_n
equal

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned
is_permutation
is_sorted
is_sorted_until

lexicographical_compare
make_heap

max_element
merge

min_element

minmax_element
mismatch
move
move_backward
next_permutation
none_of
nth_element

partial_sort
partial_sort_copy
partial_sum
partition
partition_copy

prev_permutation

reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy
sample
search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union
shuffle

sort
sort_heap

stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy

Algorithms: permutation

accumulate
adjacent_difference
adjacent_find
all_of
any_of

copy
copy_backward
copy_if
copy_n
count
count_if
destroy

destroy_n
equal

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge

iota
is_heap
is_heap_until
is_partitioned
is_permutation
is_sorted
is_sorted_until

lexicographical_compare

max_element
merge

min_element

minmax_element
mismatch
move

move_backward

none_of
nth_element

partial_sort
partial_sort_copy
partial_sum
partition
partition_copy

prev_permutation

reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy
sample
search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union
shuffle

sort

stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy

Algorithms: overlapping

accumulate
adjacent_difference
adjacent_find
all_of
any_of

copy
copy_backward

copy_if
copy_n
count
count_if
destroy

destroy_n
equal

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned

is_sorted
is_sorted_until

lexicographical_compare

max_element
merge

min_element

minmax_element
mismatch
move

move_backward

none_of
nth_element

partial_sort
partial_sort_copy
partial_sum
partition
partition_copy

reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy
sample
search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union
shuffle

sort

stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy

Algorithms: renamed

accumulate

adjacent_difference
adjacent_find
all_of
any_of

copy

copy_if
copy_n
count
count_if
destroy

destroy_n
equal

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned

is_sorted
is_sorted_until

lexicographical_compare

max_element
merge

min_element

minmax_element
mismatch
move

none_of
nth_element

partial_sort
partial_sort_copy
partition
partition_copy

partial_sum

reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy
sample
search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union
shuffle

sort

stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy

Algorithms: oddballs

adjacent_difference
adjacent_find
all_of
any_of

copy

copy_if
copy_n
count
count_if
destroy

destroy_n
equal

exclusive_scan
fill
fill_n
find
find_end
find_first_of
find_if
find_if_not
for_each
for_each_n

generate

generate_n
includes
inclusive_scan
inner_product
inplace_merge
iota
is_heap
is_heap_until
is_partitioned

is_sorted
is_sorted_until

lexicographical_compare

max_element
merge

min_element

minmax_element
mismatch
move

none_of
nth_element

partial_sort
partial_sort_copy

partition
partition_copy

reduce
remove
remove_copy
remove_copy_if
remove_if
replace
replace_copy
replace_copy_if
replace_if
reverse
reverse_copy
rotate
rotate_copy

sample

search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union

shuffle

sort

stable_partition
stable_sort
swap_ranges
transform
transform_exclusive_scan
transform_inclusive_scan
transform_reduce
uninitialized_copy
uninitialized_copy_n
uninitialized_default_construct
uninitialized_default_construct_n
uninitialized_fill
uninitialized_fill_n
uninitialized_move
uninitialized_move_n
uninitialized_value_construct
uninitialized_value_construct_n
unique
unique_copy

Supported Algorithms

adjacent_difference	generate_n	partial_sort	
adjacent_find	includes	partial_sort_copy	
all_of	inclusive_scan		
any_of	inner_product	partition	sort
	inplace_merge	partition_copy	
			stable_partition
copy	is_heap		stable_sort
	is_heap_until		swap_ranges
	is_partitioned		transform
copy_if		reduce	transform_exclusive_scan
copy_n	is_sorted	remove	transform_inclusive_scan
count	is_sorted_until	remove_copy	transform_reduce
count_if		remove_copy_if	uninitialized_copy
destroy		remove_if	uninitialized_copy_n
	lexicographical_compare	replace	uninitialized_default_construct
destroy_n		replace_copy	uninitialized_default_construct_n
equal		replace_copy_if	uninitialized_fill
		replace_if	uninitialized_fill_n
exclusive_scan	max_element	reverse	uninitialized_move
fill	merge	reverse_copy	uninitialized_move_n
fill_n		rotate	uninitialized_value_construct
find	min_element	rotate_copy	uninitialized_value_construct_n
find_end			unique
find_first_of	minmax_element	search	unique_copy
find_if	mismatch	search_n	
find_if_not	move	set_difference	
for_each		set_intersection	
for_each_n		set_symmetric_difference	
		set_union	
generate	none_of		
	nth_element		

Algorithms: map

adjacent_difference
adjacent_find
all_of
any_of

copy

copy_if
copy_n

count

count_if

destroy

destroy_n

equal

exclusive_scan

fill

fill_n

find

find_end

find_first_of

find_if

find_if_not

for_each

for_each_n

generate

generate_n

includes

inclusive_scan

inner_product

inplace_merge

is_heap

is_heap_until

is_partitioned

is_sorted

is_sorted_until

lexicographical_compare

max_element

merge

min_element

minmax_element

mismatch

move

none_of

nth_element

partial_sort

partial_sort_copy

partition

partition_copy

reduce

remove

remove_copy

remove_copy_if

remove_if

replace

replace_copy

replace_copy_if

replace_if

reverse

reverse_copy

rotate

rotate_copy

search

search_n

set_difference

set_intersection

set_symmetric_difference

set_union

sort

stable_partition

stable_sort

swap_ranges

transform

transform_exclusive_scan

transform_inclusive_scan

transform_reduce

uninitialized_*

unique

unique_copy

Additional Constraints

- `for_each()`, `for_each_n()` don't return the function
- `copy()`, `move()`: source and range can't overlap
- `copy_n()` can overlap: probably a defect
- for non-random access: may require reduce

Algorithms: reduce

adjacent_difference
adjacent_find
all_of
any_of

copy_if
count
count_if

equal

exclusive_scan
find
find_end
find_first_of
find_if
find_if_not

includes
inclusive_scan
inner_product
inplace_merge
is_heap
is_heap_until
is_partitioned
is_sorted
is_sorted_until

lexicographical_compare

max_element

merge
min_element
minmax_element
mismatch

none_of
nth_element

partial_sort
partial_sort_copy

partition
partition_copy

reduce
remove
remove_copy
remove_copy_if
remove_if

rotate
rotate_copy

search
search_n
set_difference
set_intersection
set_symmetric_difference
set_union

sort

stable_partition
stable_sort

transform_exclusive_scan
transform_inclusive_scan
transform_reduce

unique
unique_copy

Algorithms: reduce

- `accumulate()` becomes `reduce()`
- operations need to be associative
- find algorithms may omit using the entire range

Algorithms: scan

adjacent_difference

inclusive_scan

partial_sort
partial_sort_copy

inplace_merge

partition
partition_copy

sort

stable_partition
stable_sort

copy_if

remove
remove_copy
remove_copy_if
remove_if

transform_exclusive_scan
transform_inclusive_scan
transform_reduce

exclusive_scan

merge

rotate
rotate_copy

unique
unique_copy

set_difference
set_intersection
set_symmetric_difference
set_union

nth_element

Algorithms: scan

- `partial_sum()` becomes `inclusive_scan()`
- may produce different results when operation isn't associative
- `inclusive_scan()`: $r[i]$ uses $s[0], \dots, s[i]$
- `exclusive_scan()`: $r[i]$ uses $s[0], \dots, s[i - 1]$
- note: order of initial value and operation differ between `inclusive_scan()` and `exclusive_scan()`!

Algorithms: fused

adjacent_difference

partial_sort
partial_sort_copy

inplace_merge

partition
partition_copy

sort

stable_partition
stable_sort

copy_if

transform_exclusive_scan

remove
remove_copy
remove_copy_if
remove_if

transform_inclusive_scan

transform_reduce

merge

rotate
rotate_copy

unique
unique_copy

nth_element

set_difference
set_intersection
set_symmetric_difference
set_union

Algorithms: gather

adjacent_difference

inplace_merge

copy_if

merge

nth_element

partial_sort
partial_sort_copy

partition
partition_copy

remove
remove_copy
remove_copy_if
remove_if

rotate
rotate_copy

set_difference
set_intersection
set_symmetric_difference
set_union

sort
stable_partition
stable_sort

unique
unique_copy

Algorithms: special

adjacent_difference

partial_sort
partial_sort_copy

inplace_merge

partition

sort

stable_partition
stable_sort

merge

rotate

set_difference
set_intersection
set_symmetric_difference
set_union

nth_element

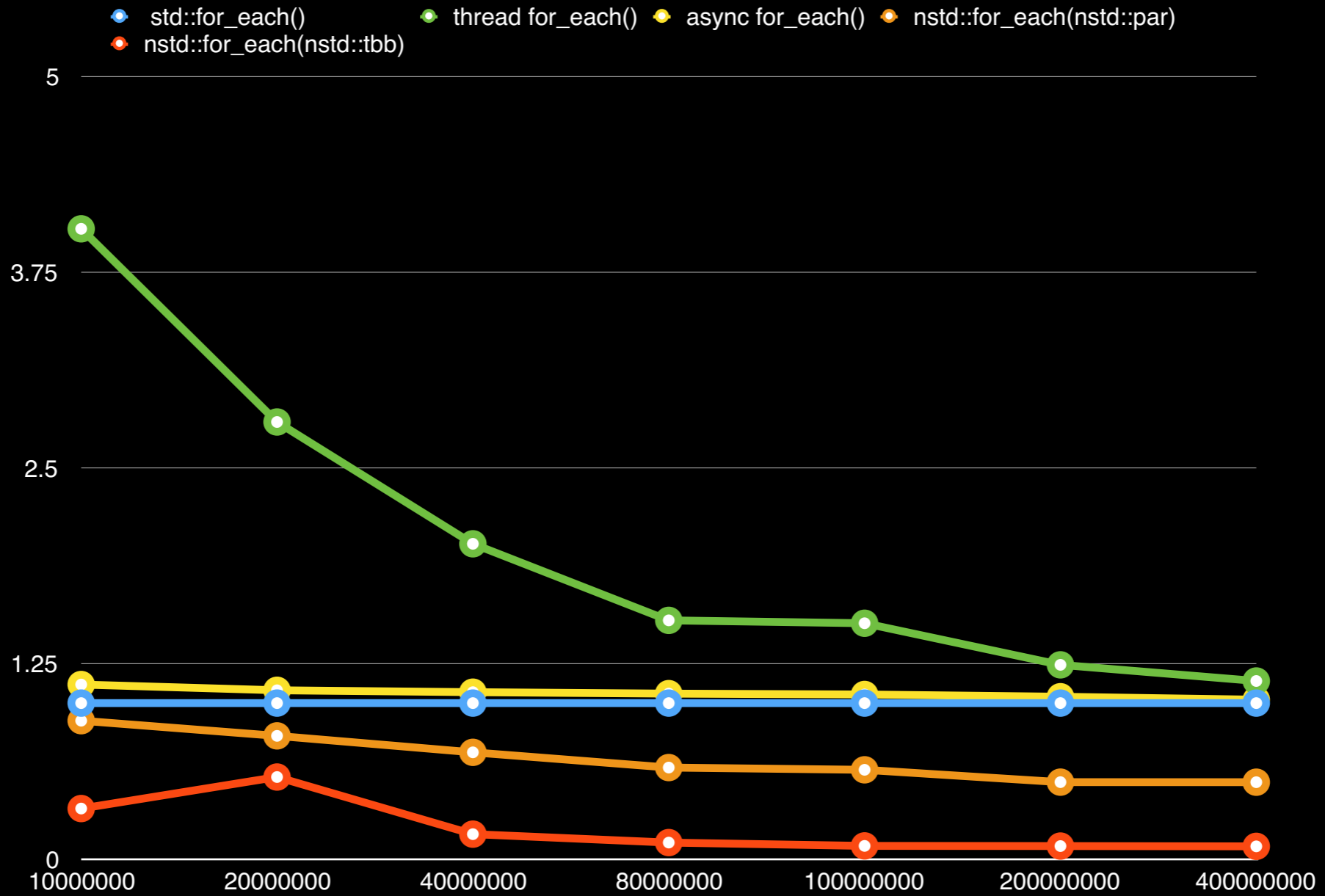
Results: Machines

- Intel Xeon Phi: 64 cores, 4 hyper threaded, 96GB
- Intel I7: 4 cores, 2x hyper threaded, 32GB
- ARM: 4 cores, not hyper threaded, 1GB

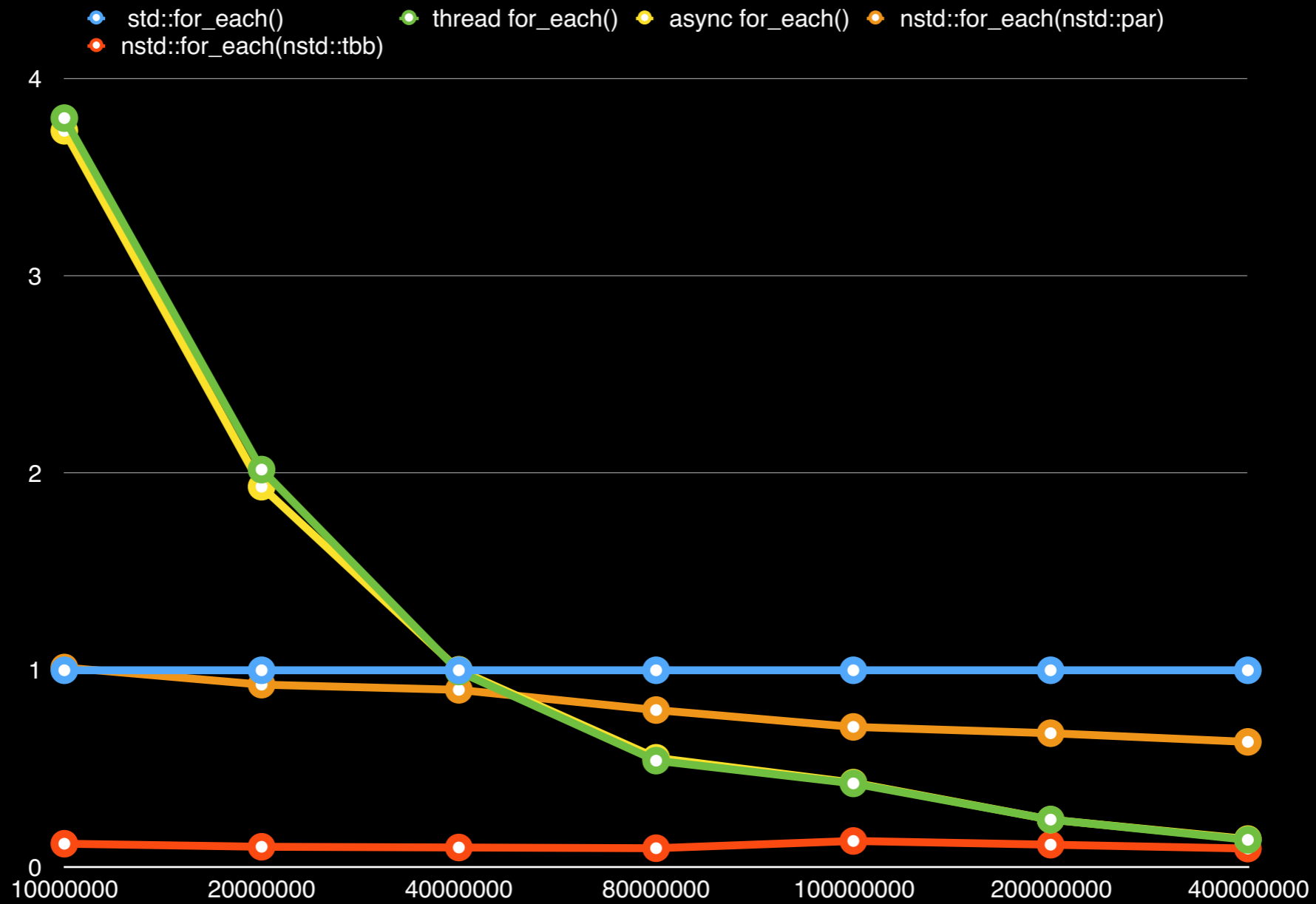
Results: map

```
for (; it != end; ++it) {  
    *it *= 17;  
}
```

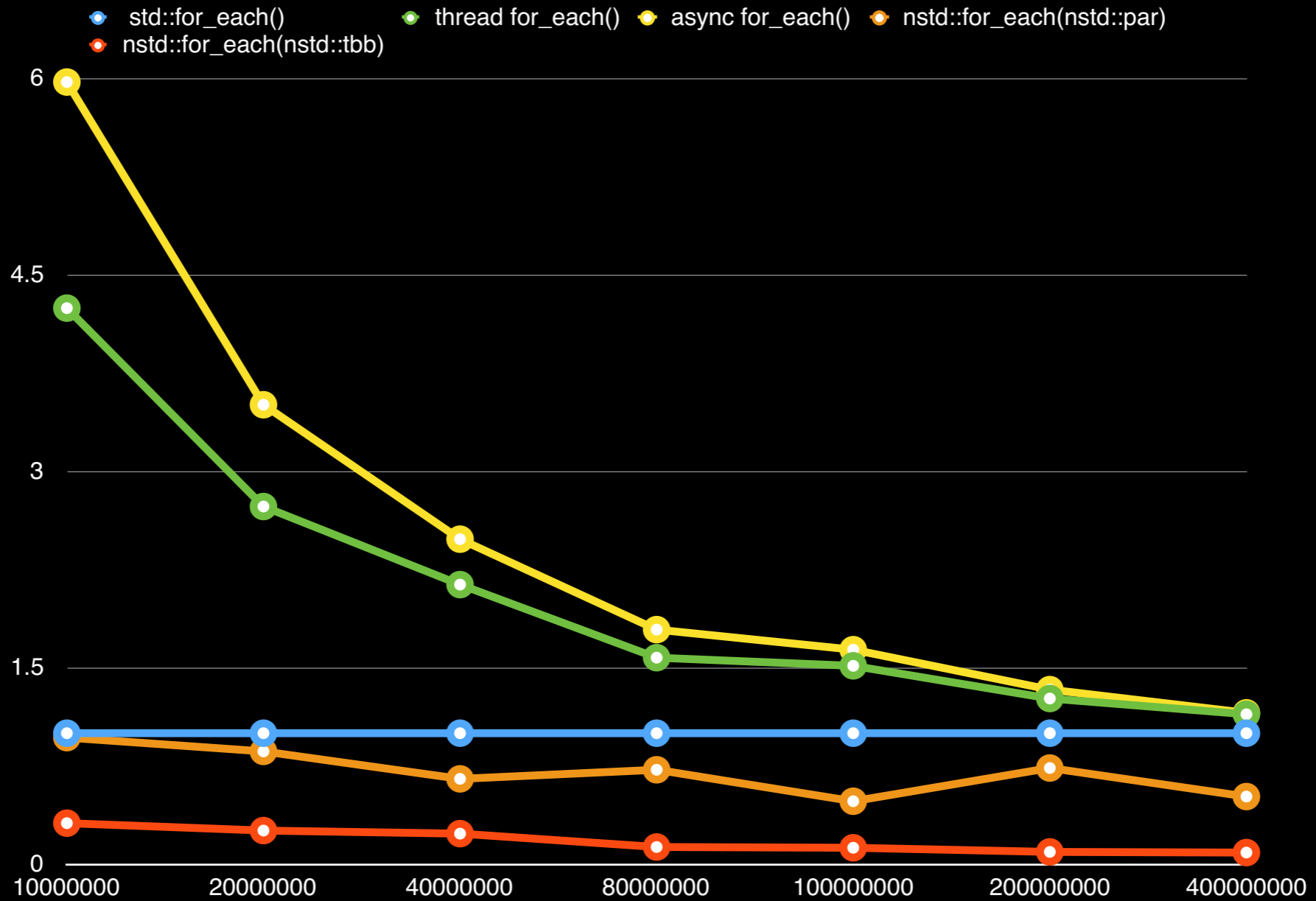
Results: map Intel phi



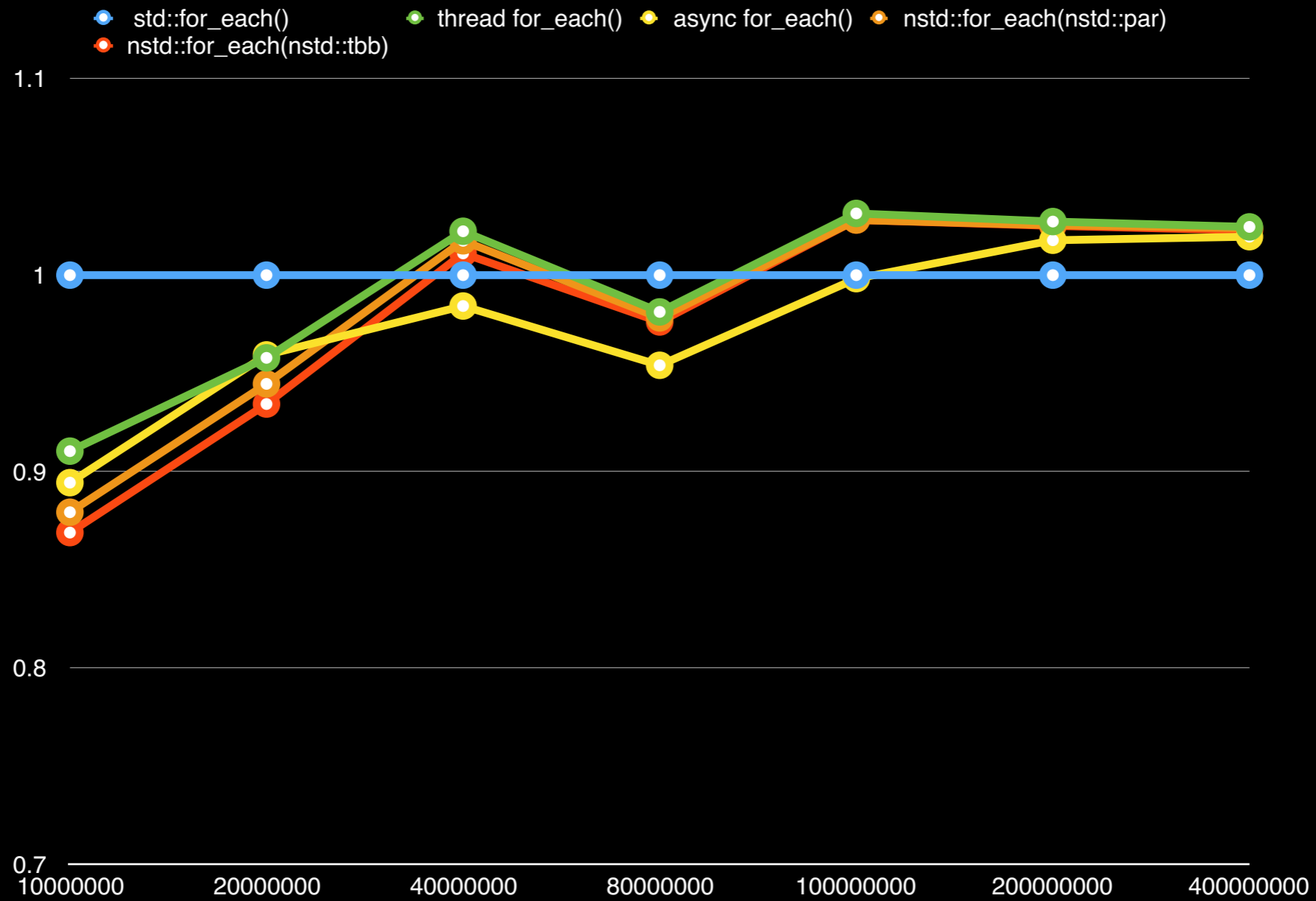
Results: map gcc phi



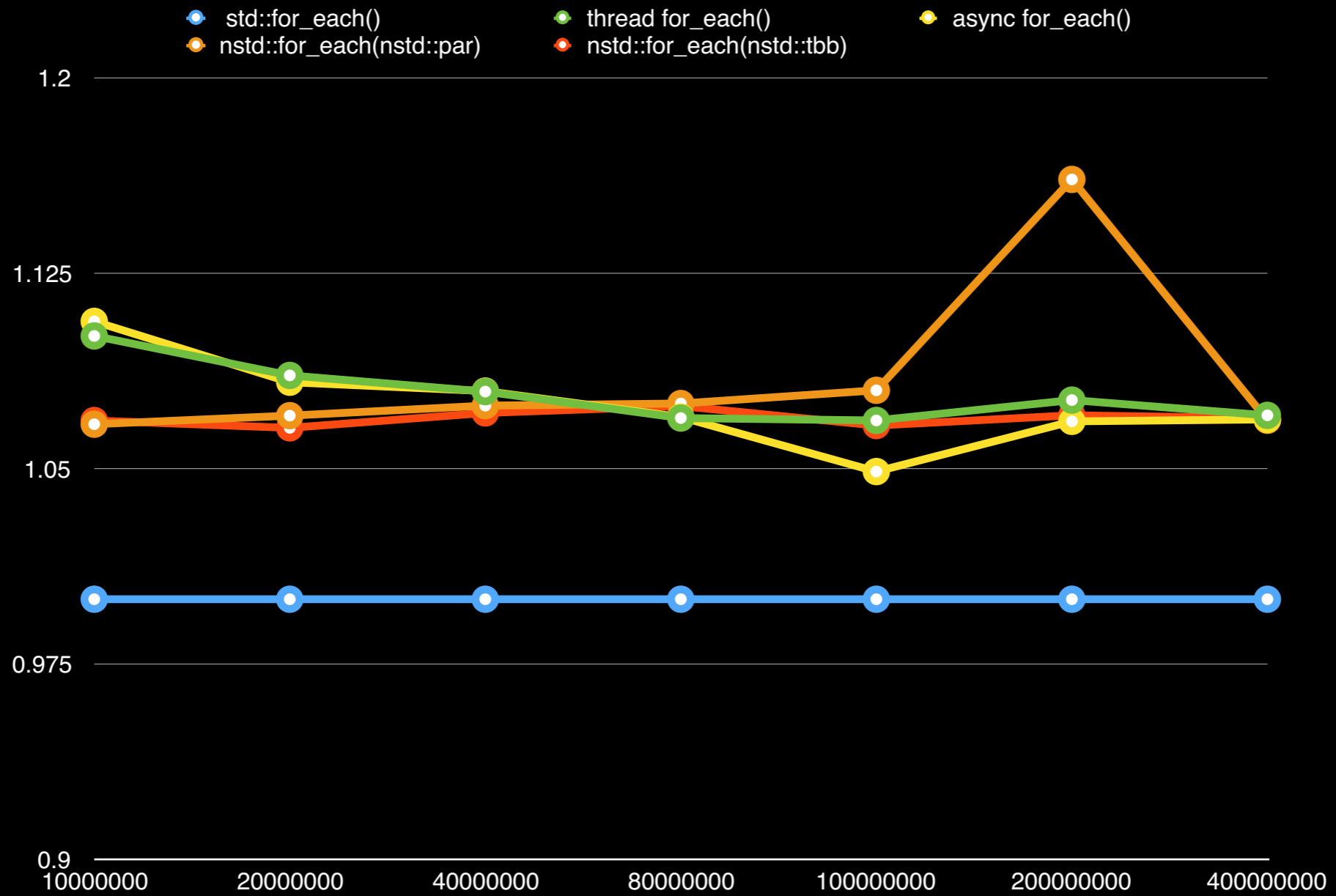
Results: map clang phi



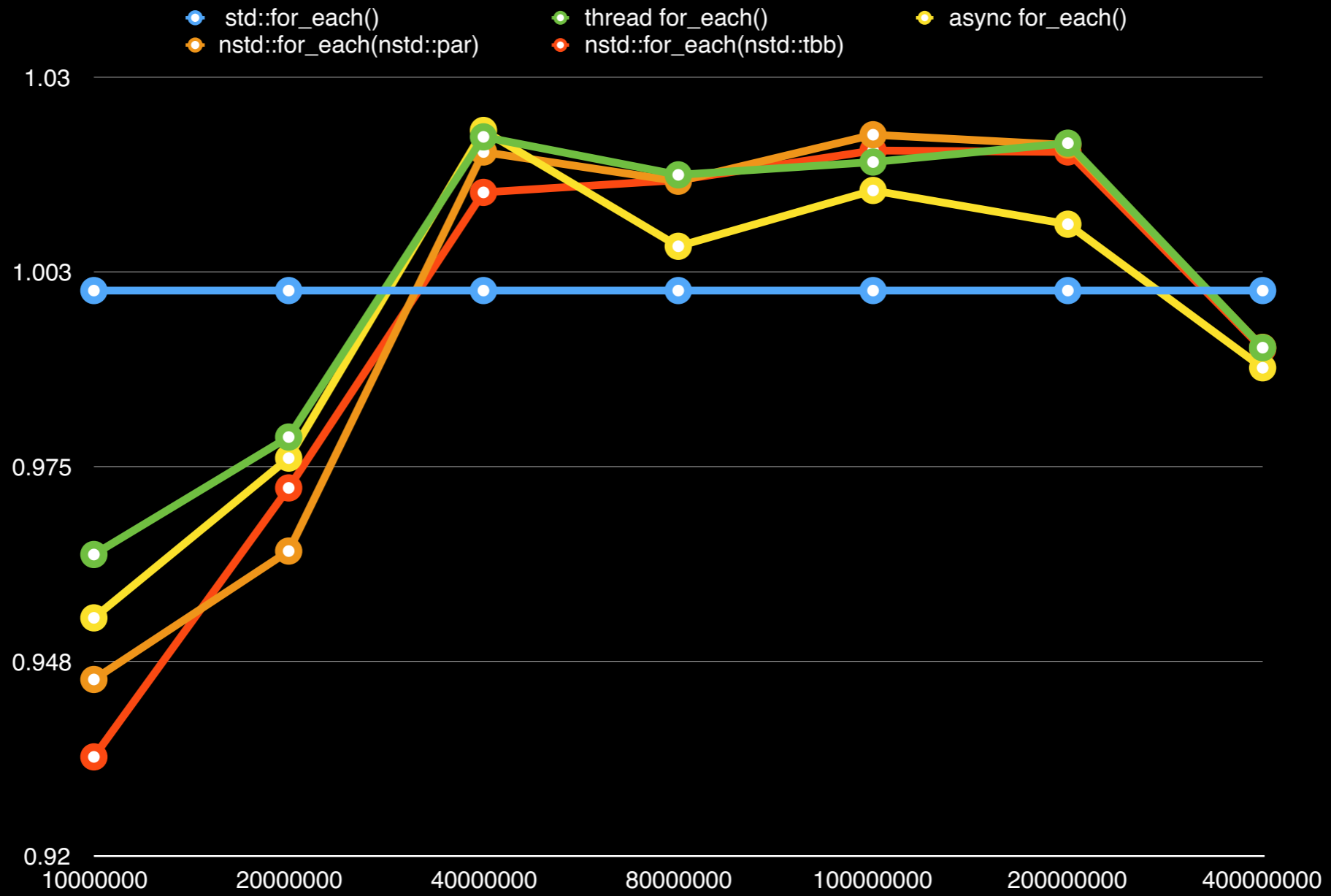
Results: map Intel I7



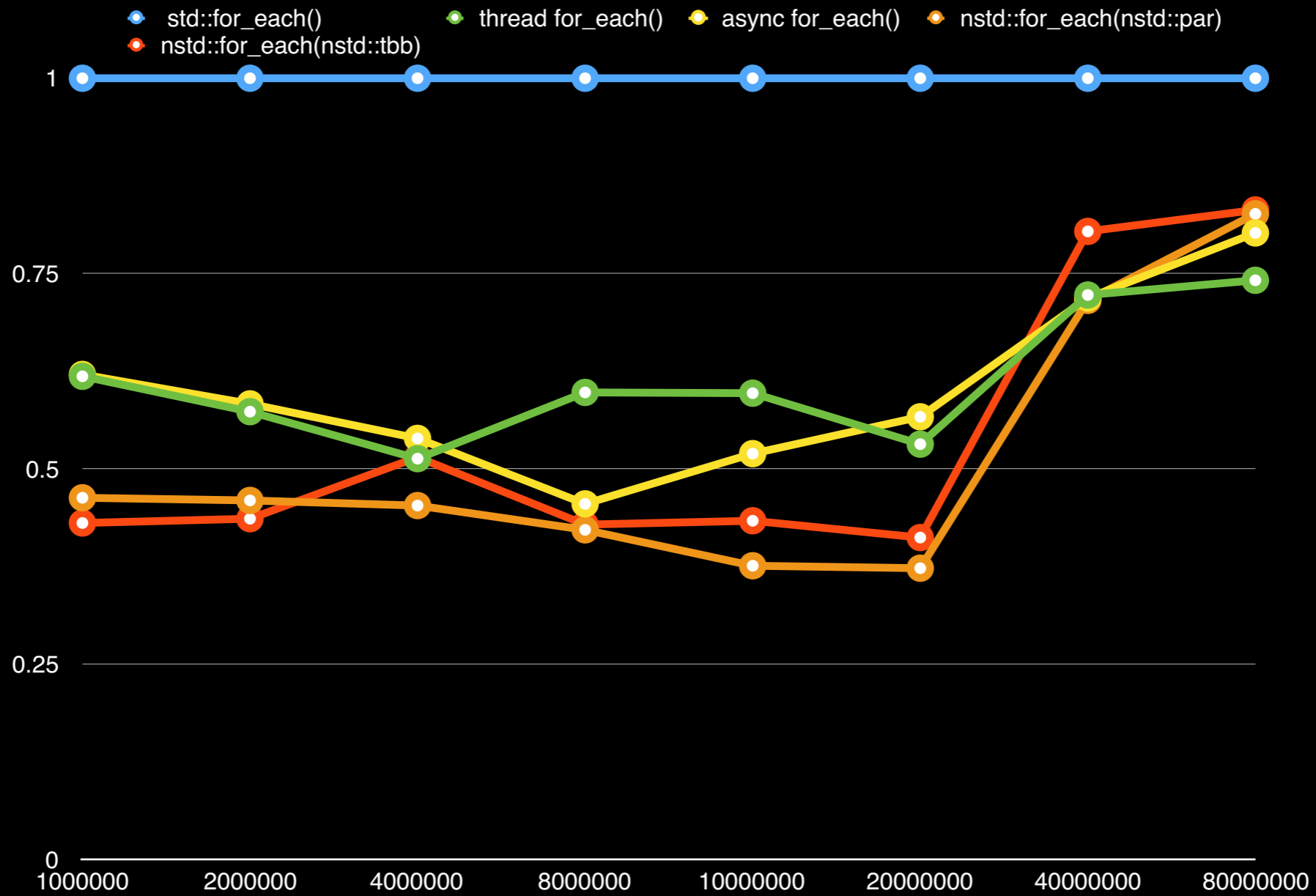
Results: map gcc 17



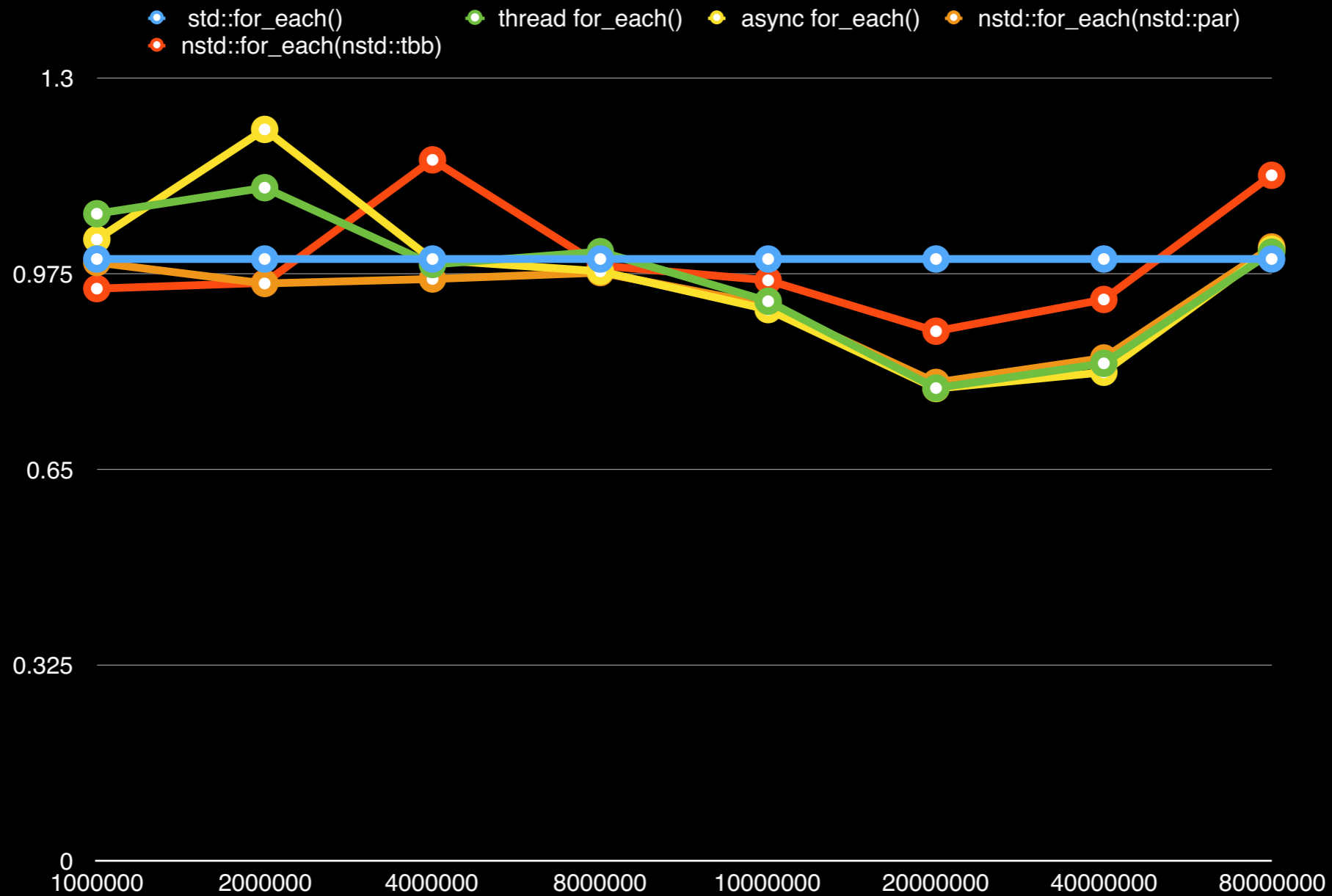
Results: map clang 17



Results: map gcc ARM



Results: map clang ARM

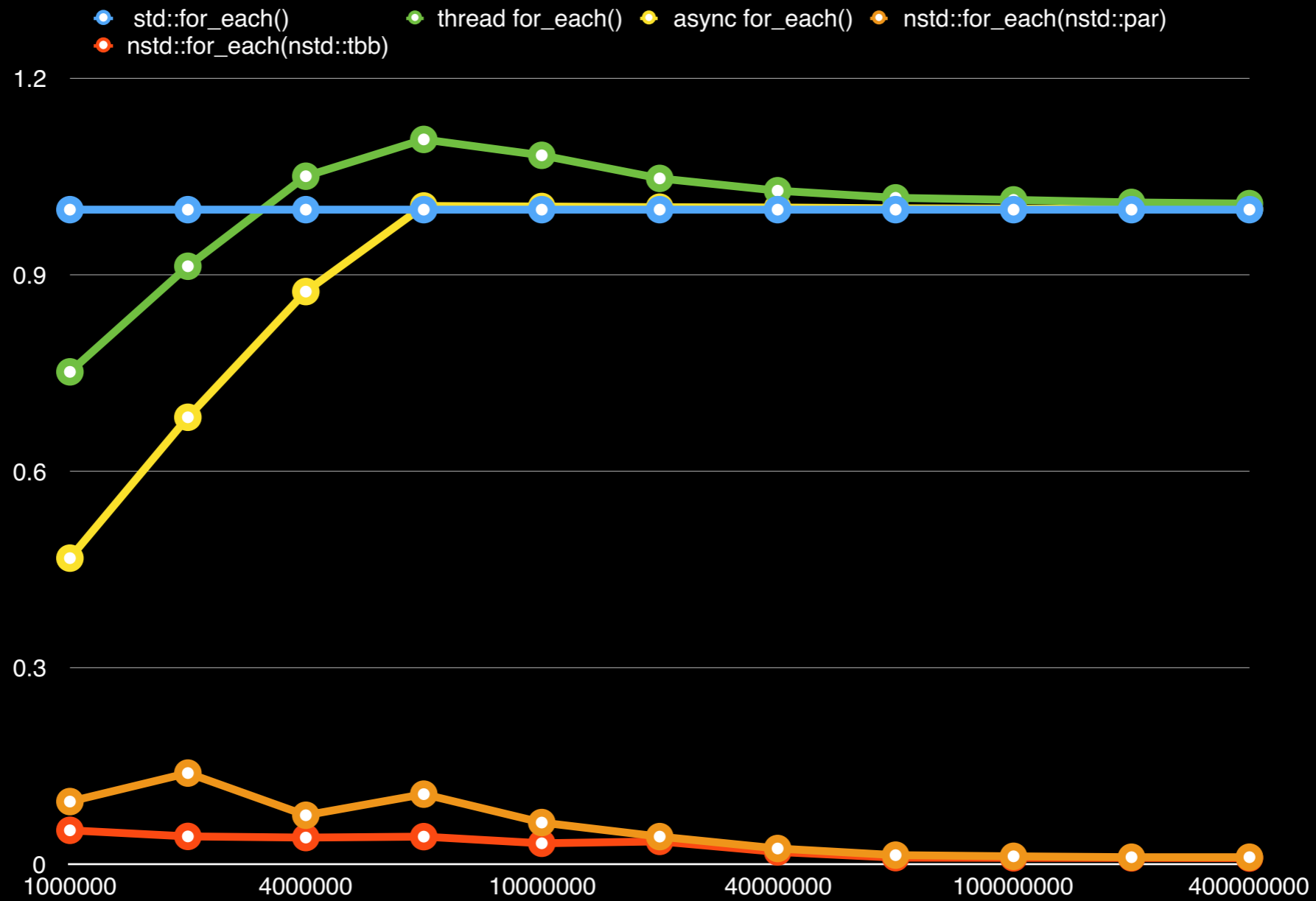


Results: map

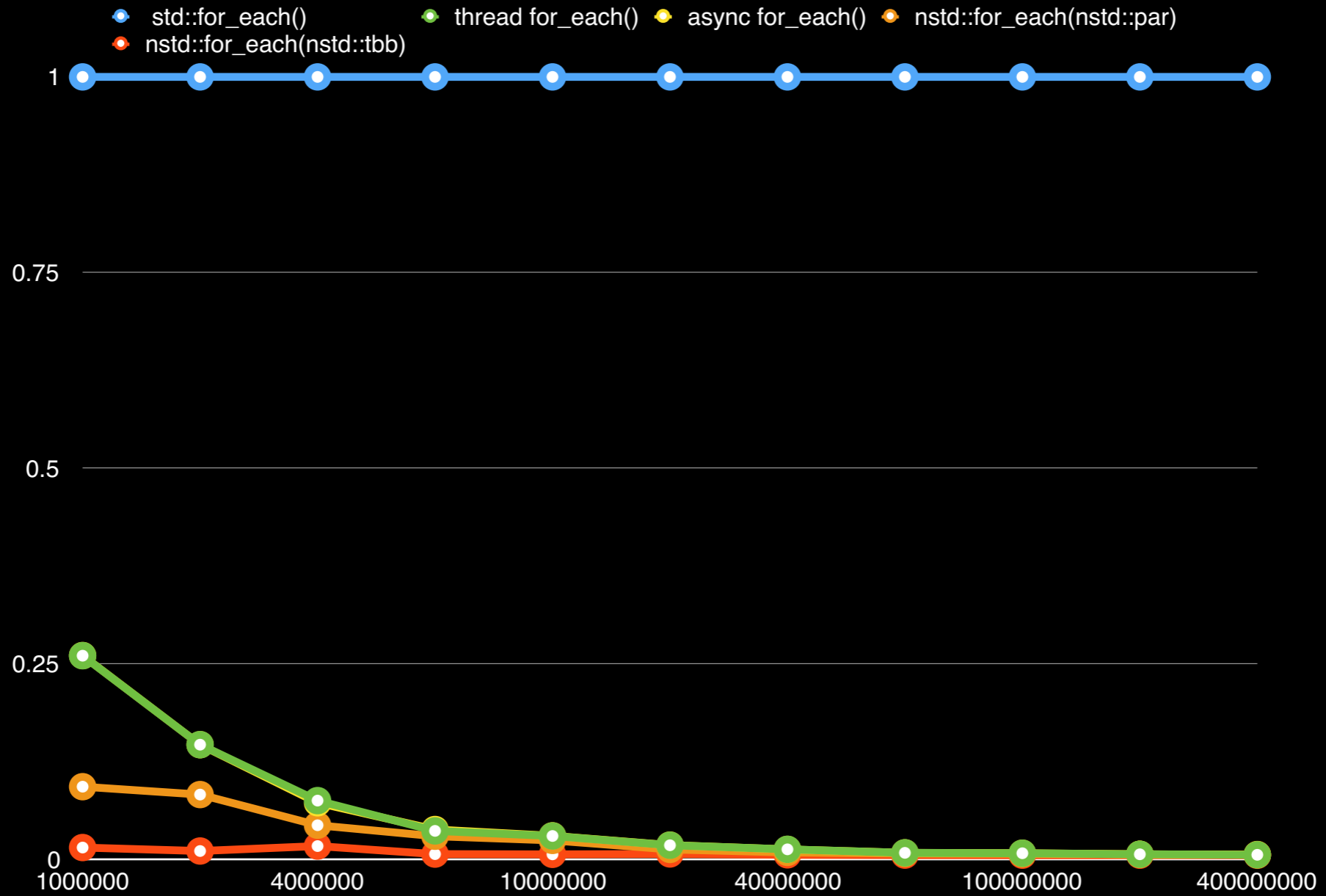
```
for (; it != end; ++it) {
    constexpr int max(2000);
    std::complex<double> p(2.5 * *it / s - 0.5, 0.001);
    int count(0);

    for (std::complex<double> v(p);
         norm(v) < 4.0 && count != max; ++count) {
        v = v * v - p;
    }
    *it = count;
}
```

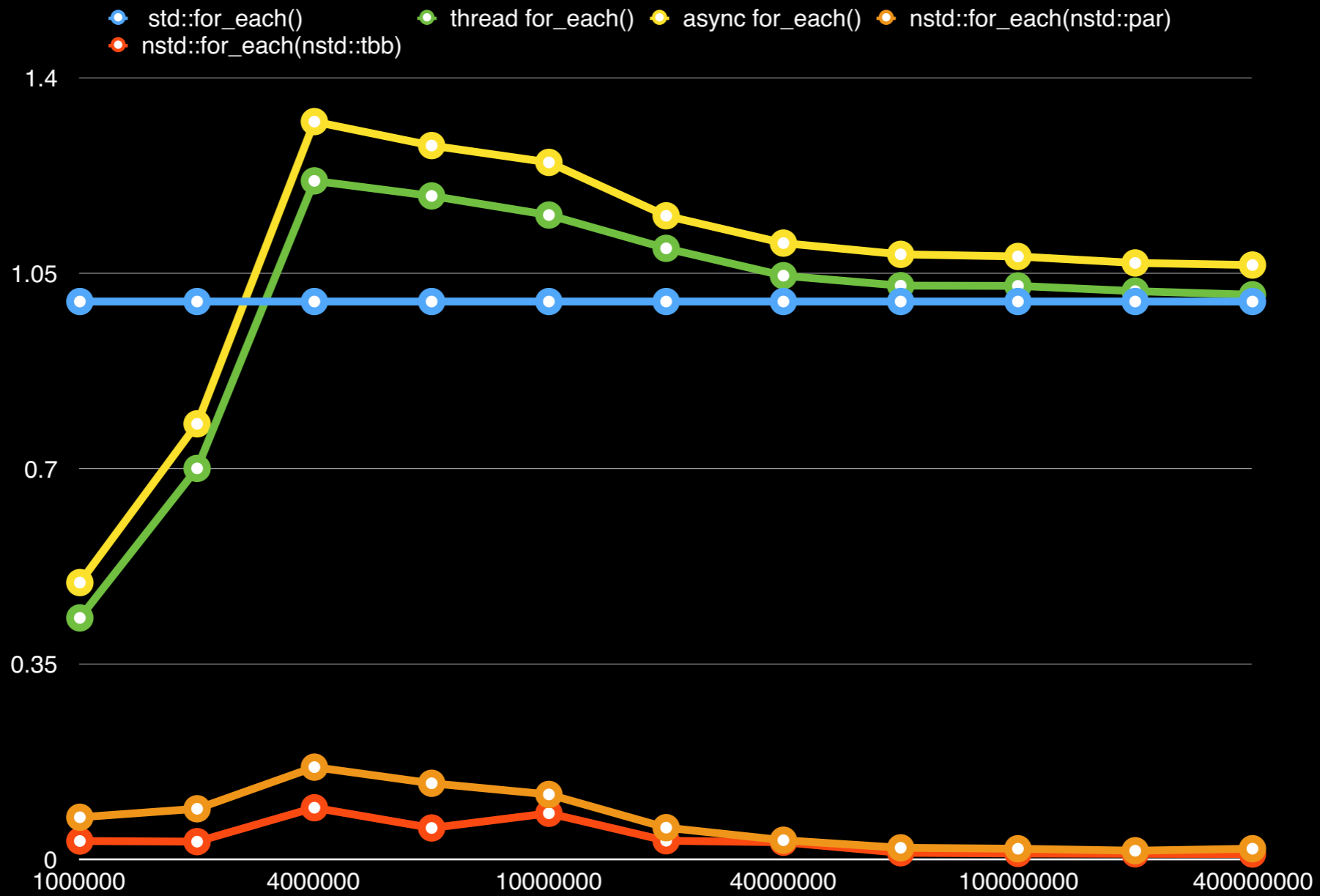
Results: work Intel phi



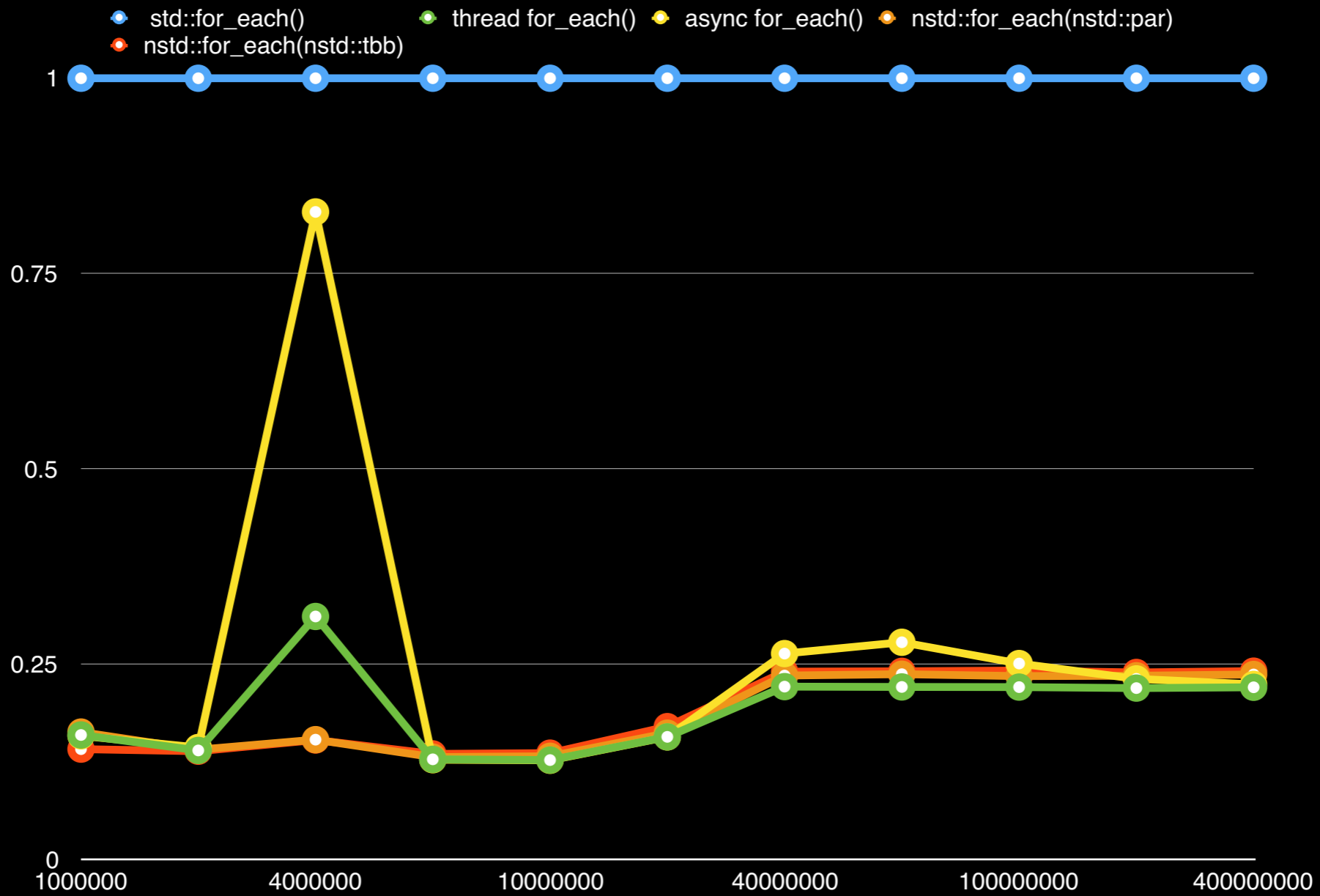
Results: work gcc phi



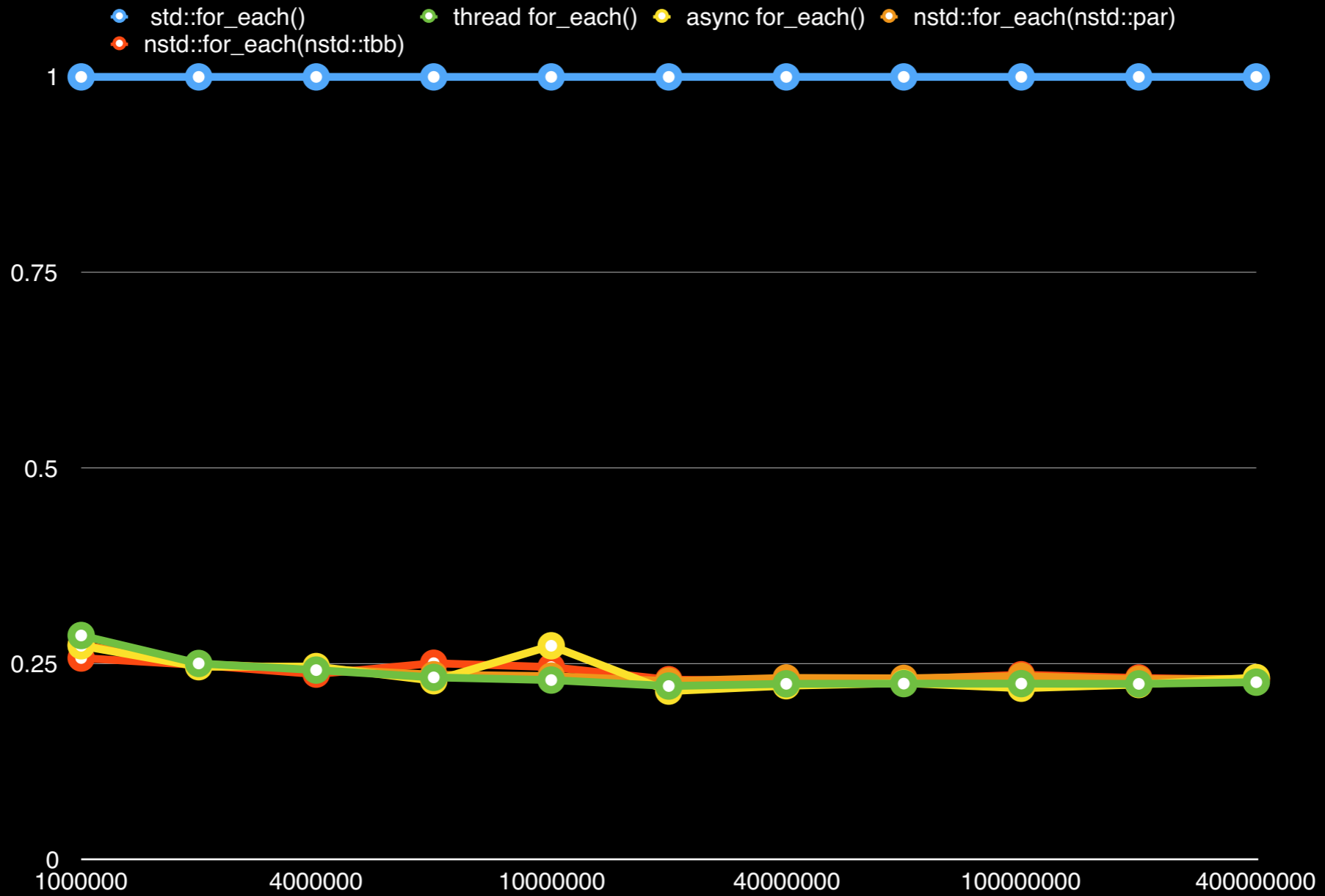
Results: work clang phi



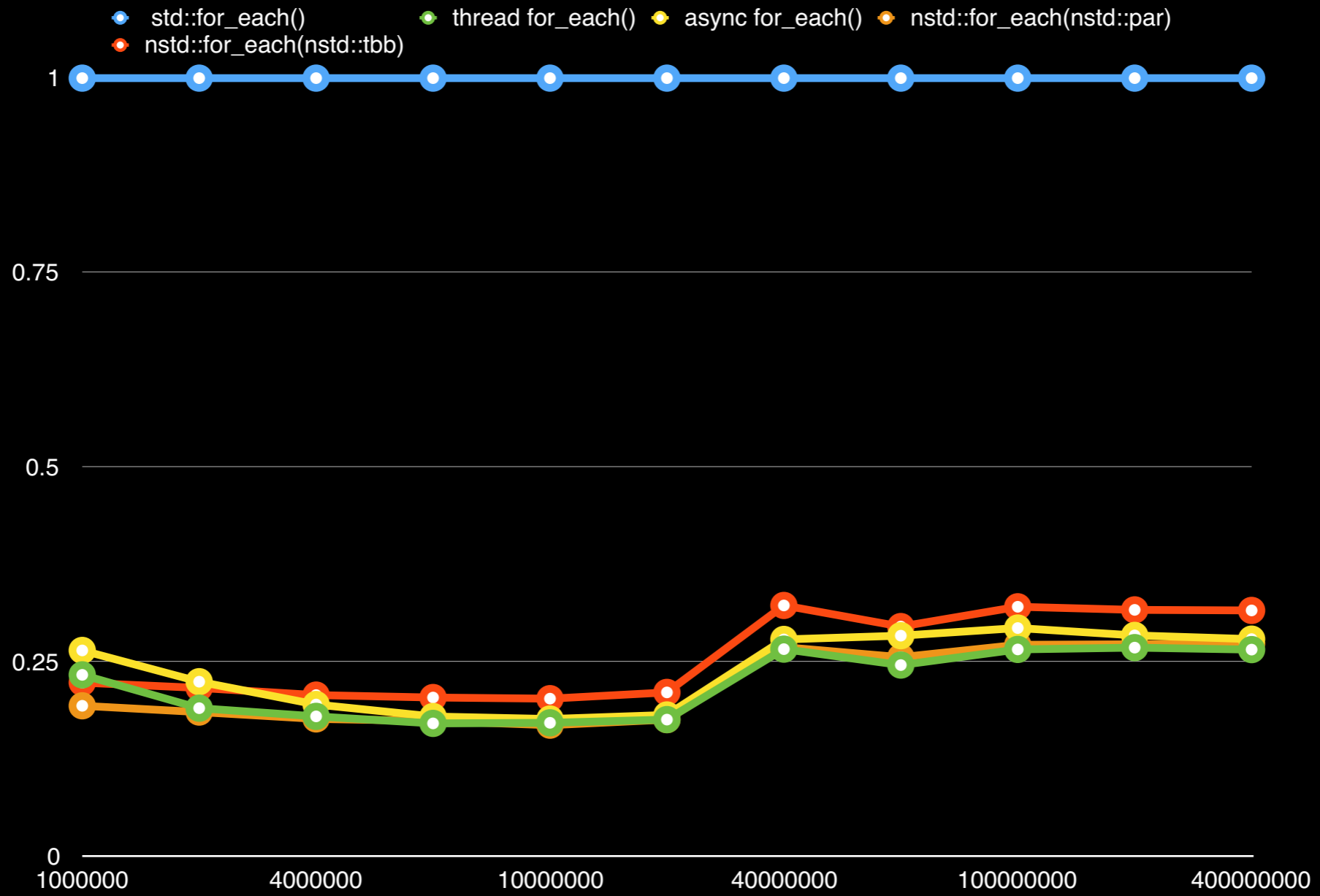
Results: work Intel I7



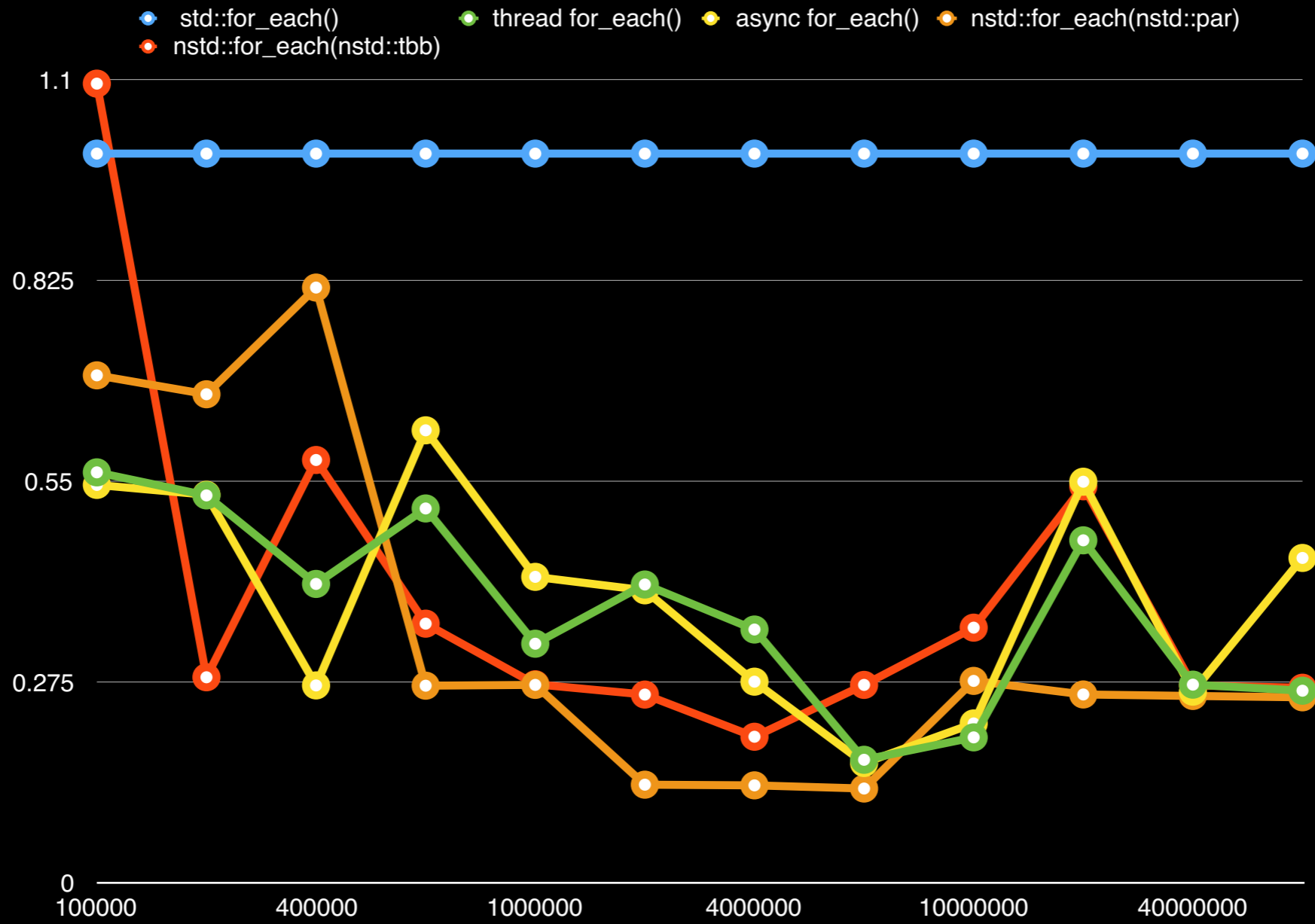
Results: work gcc 17



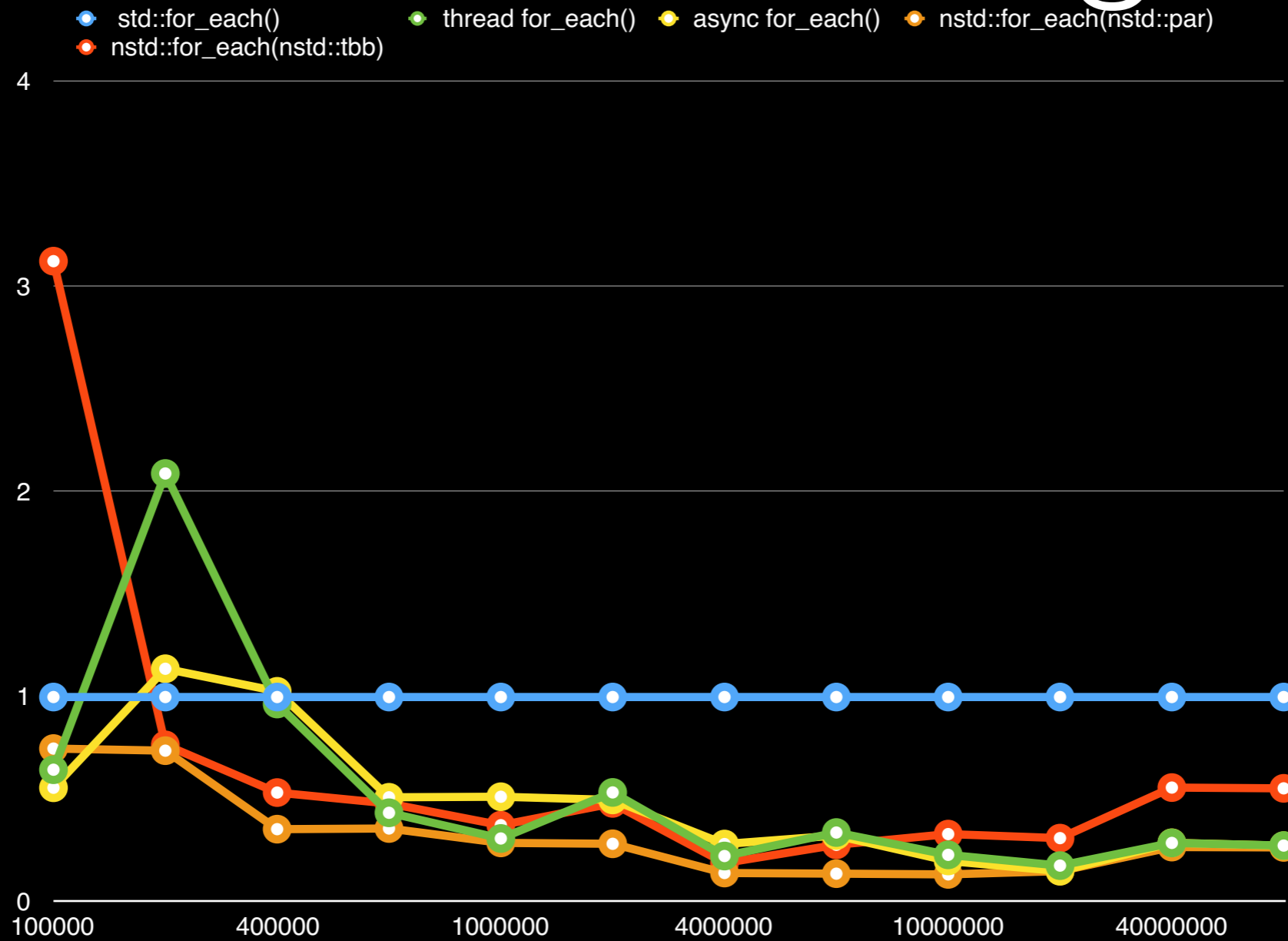
Results: work clang 17



Results: work gcc ARM



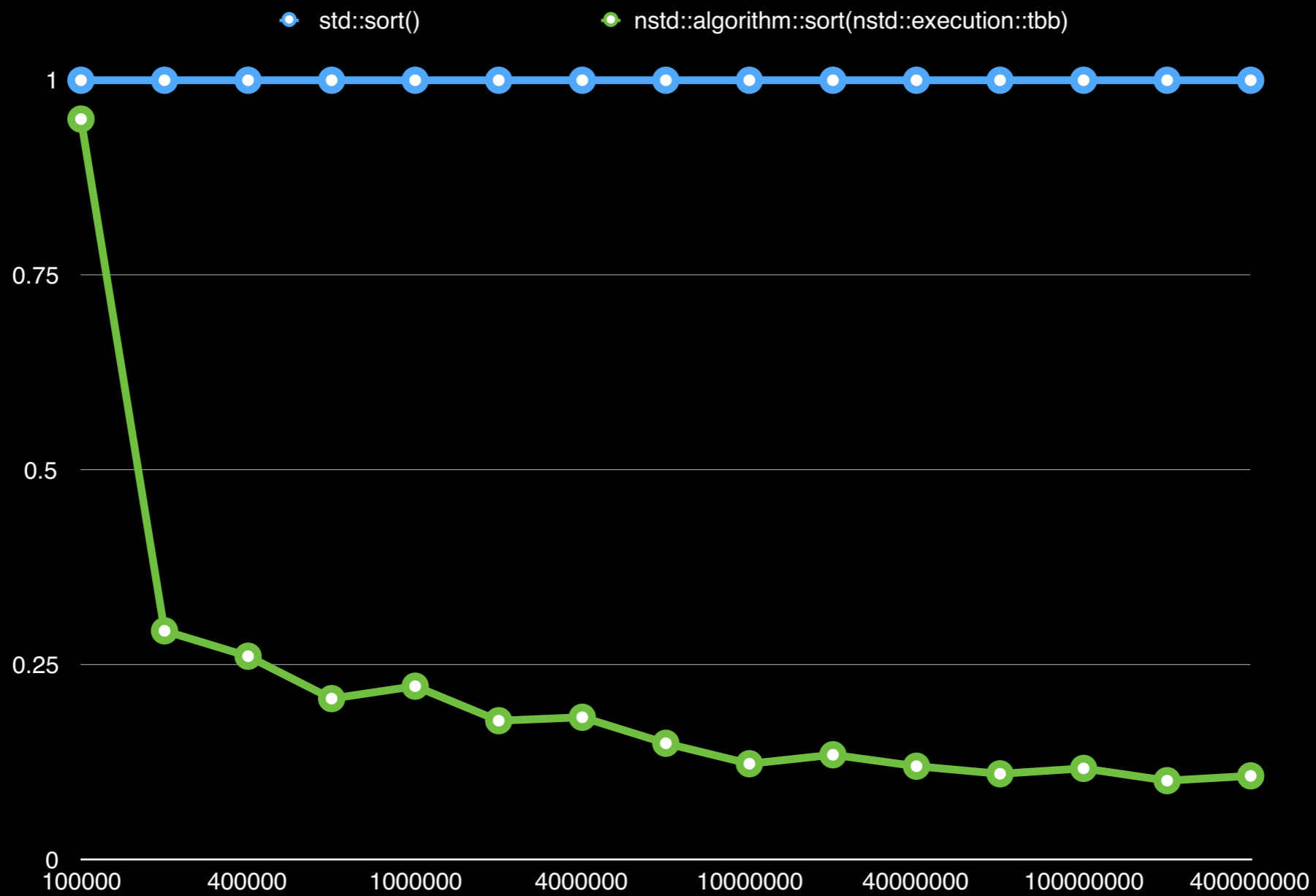
Results: work clang ARM



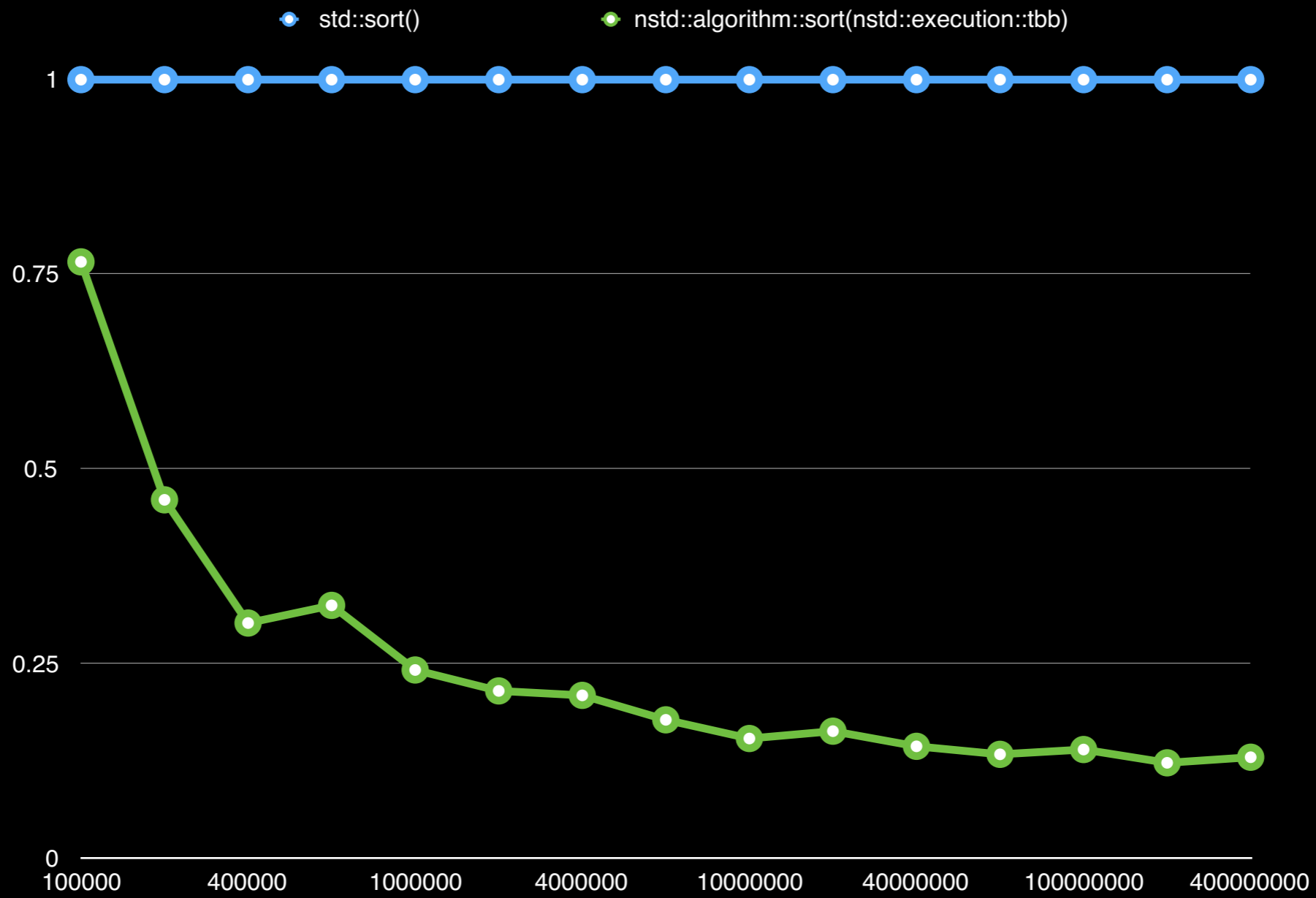
Results: sort

```
sort(begin, end);
```

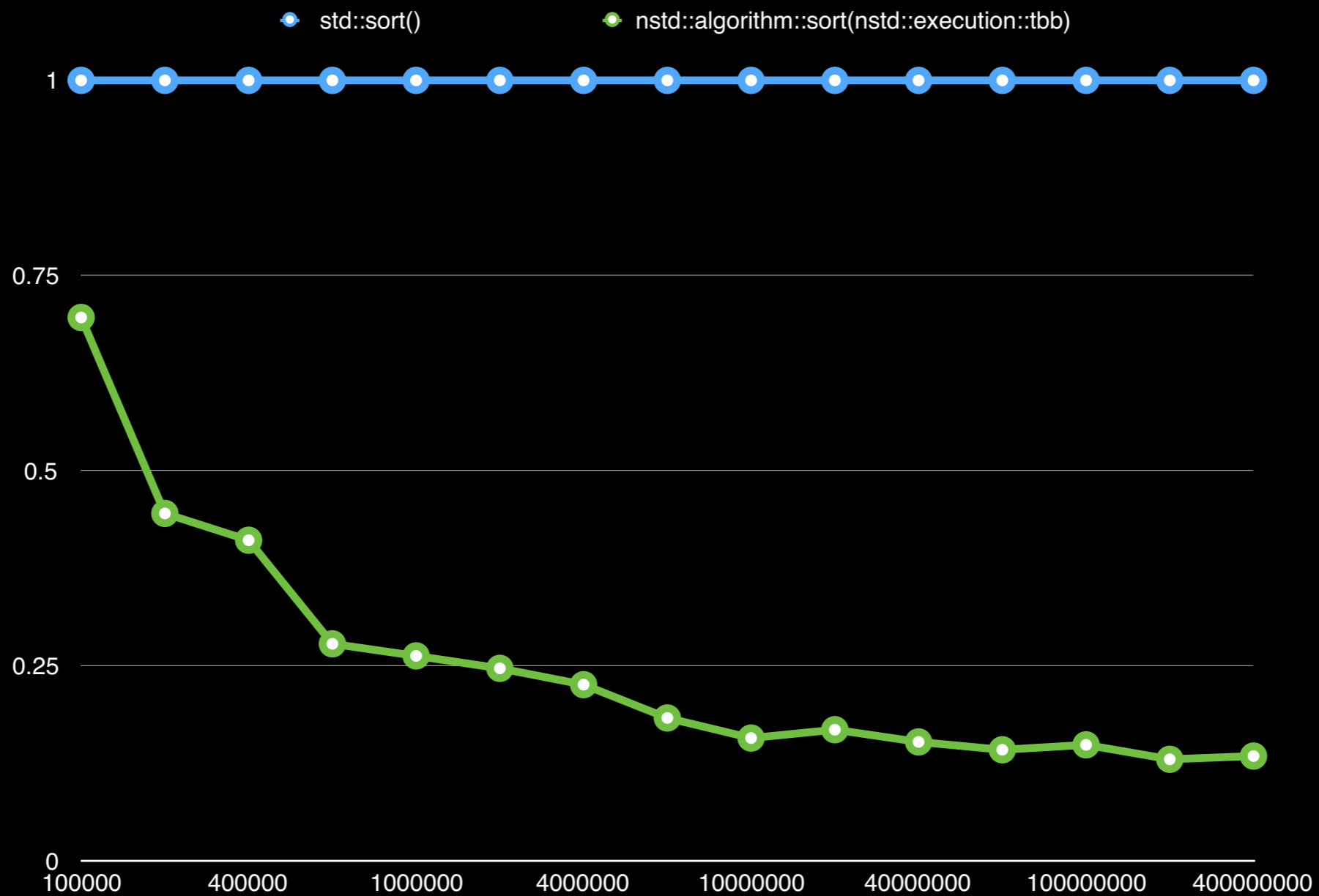
Results: sort Intel phi



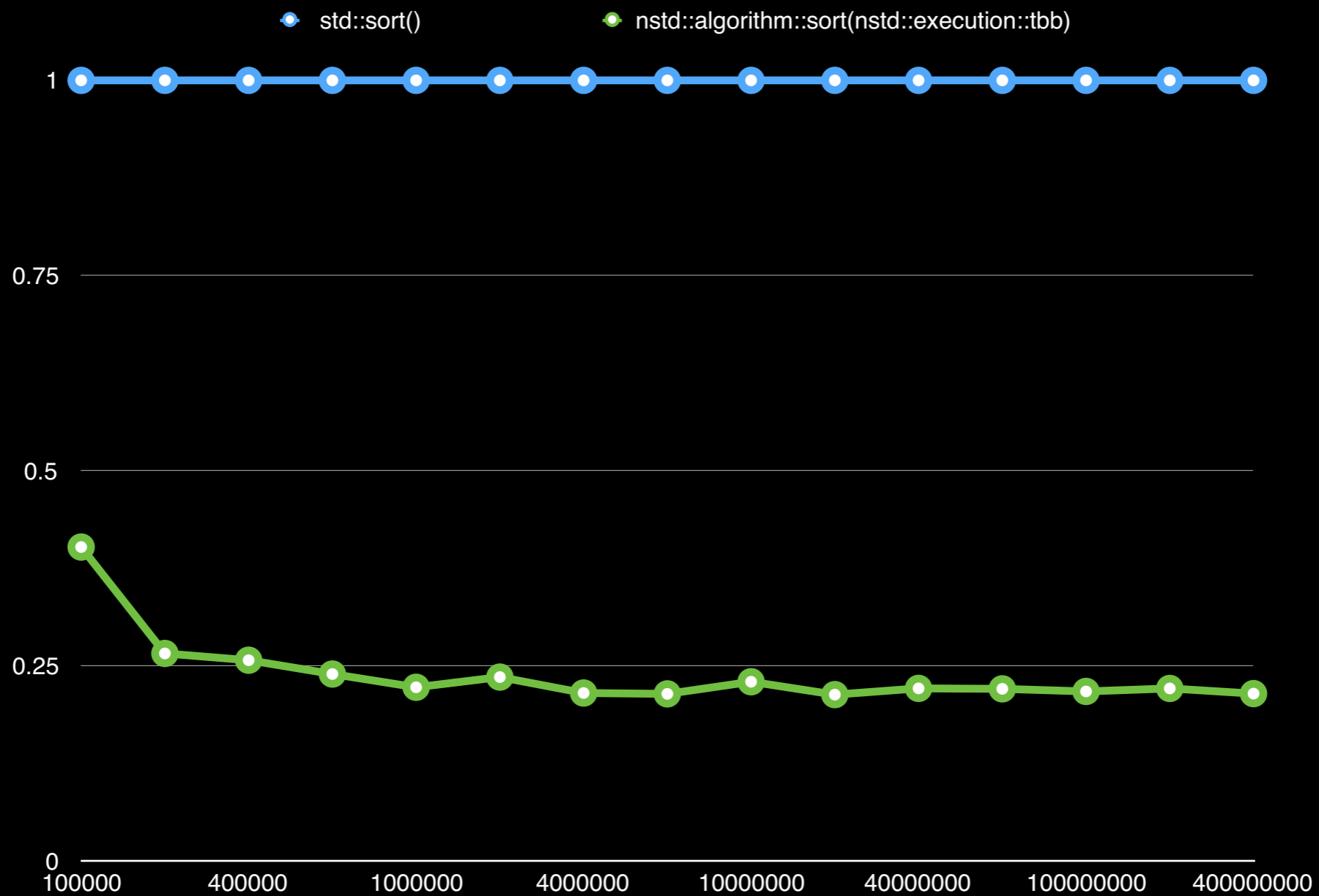
Results: sort gcc phi



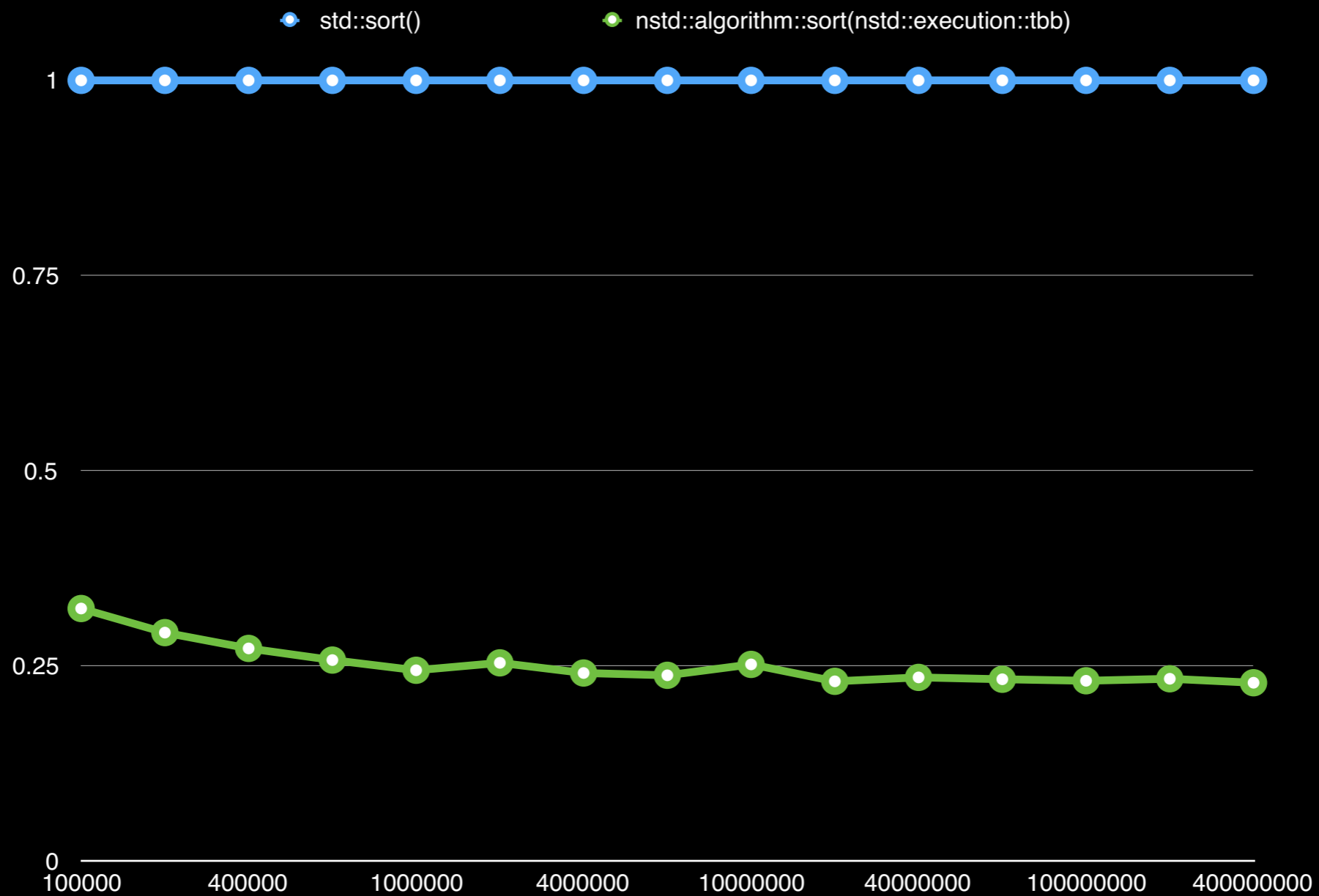
Results: sort clang phi



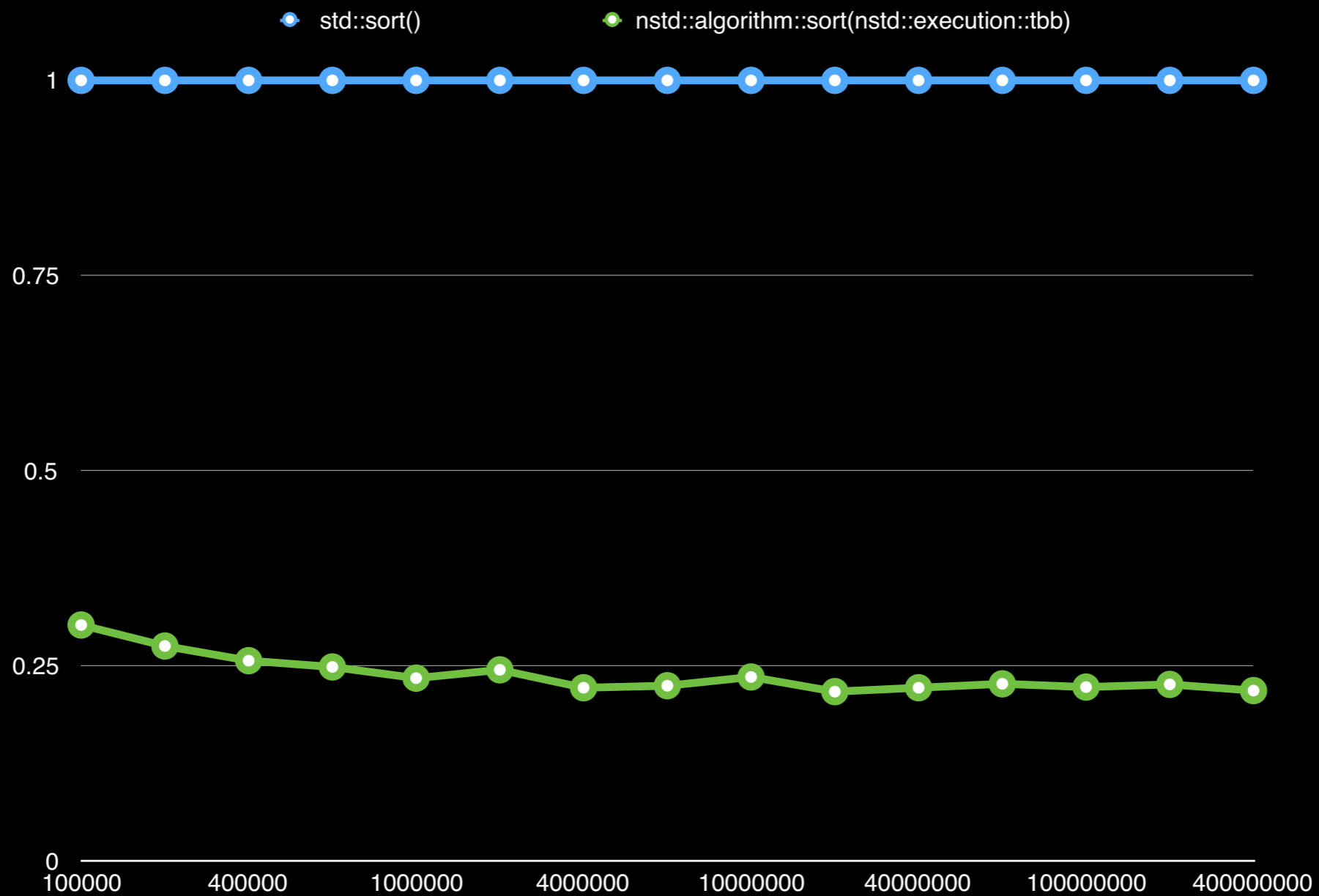
Results: sort Intel I7



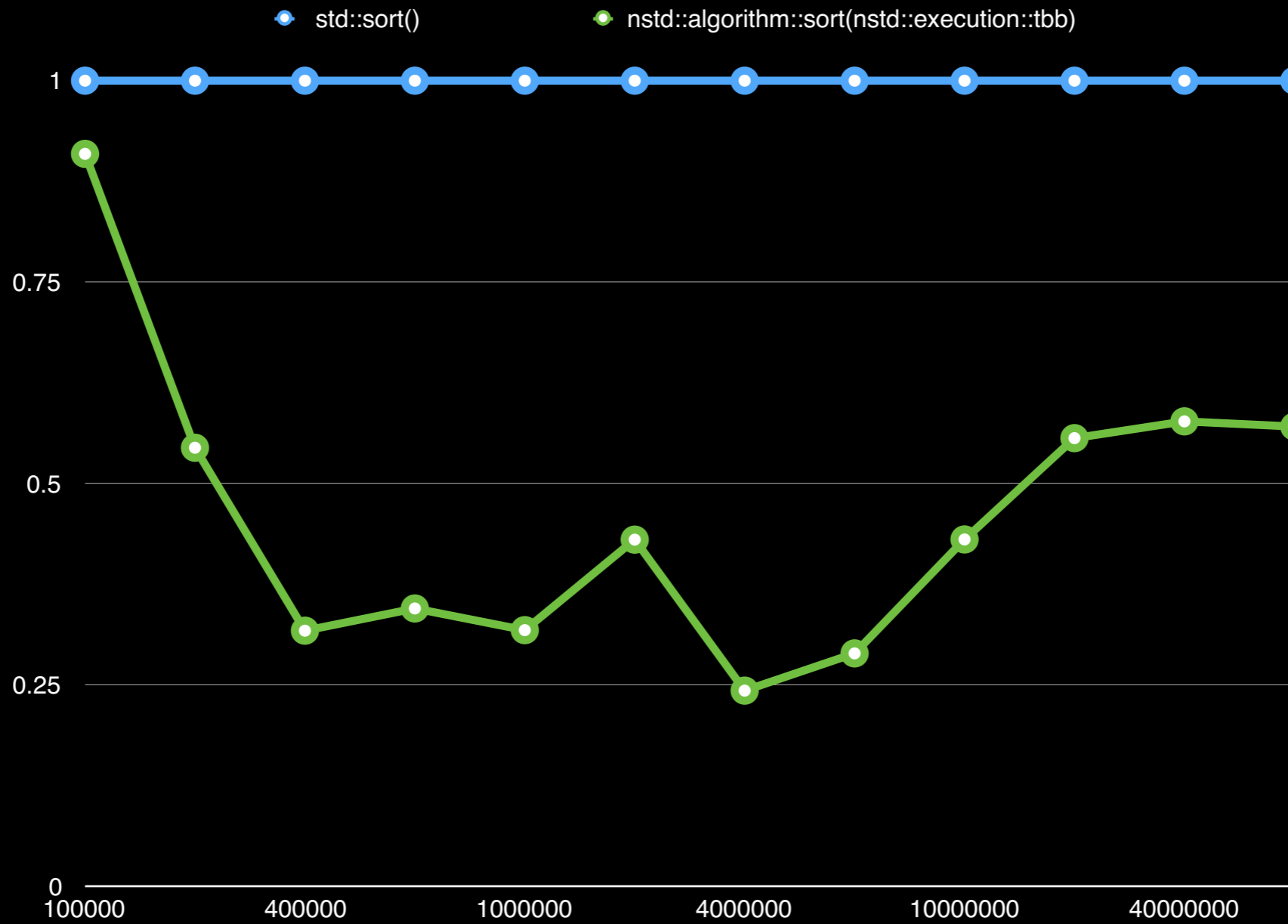
Results: sort gcc 17



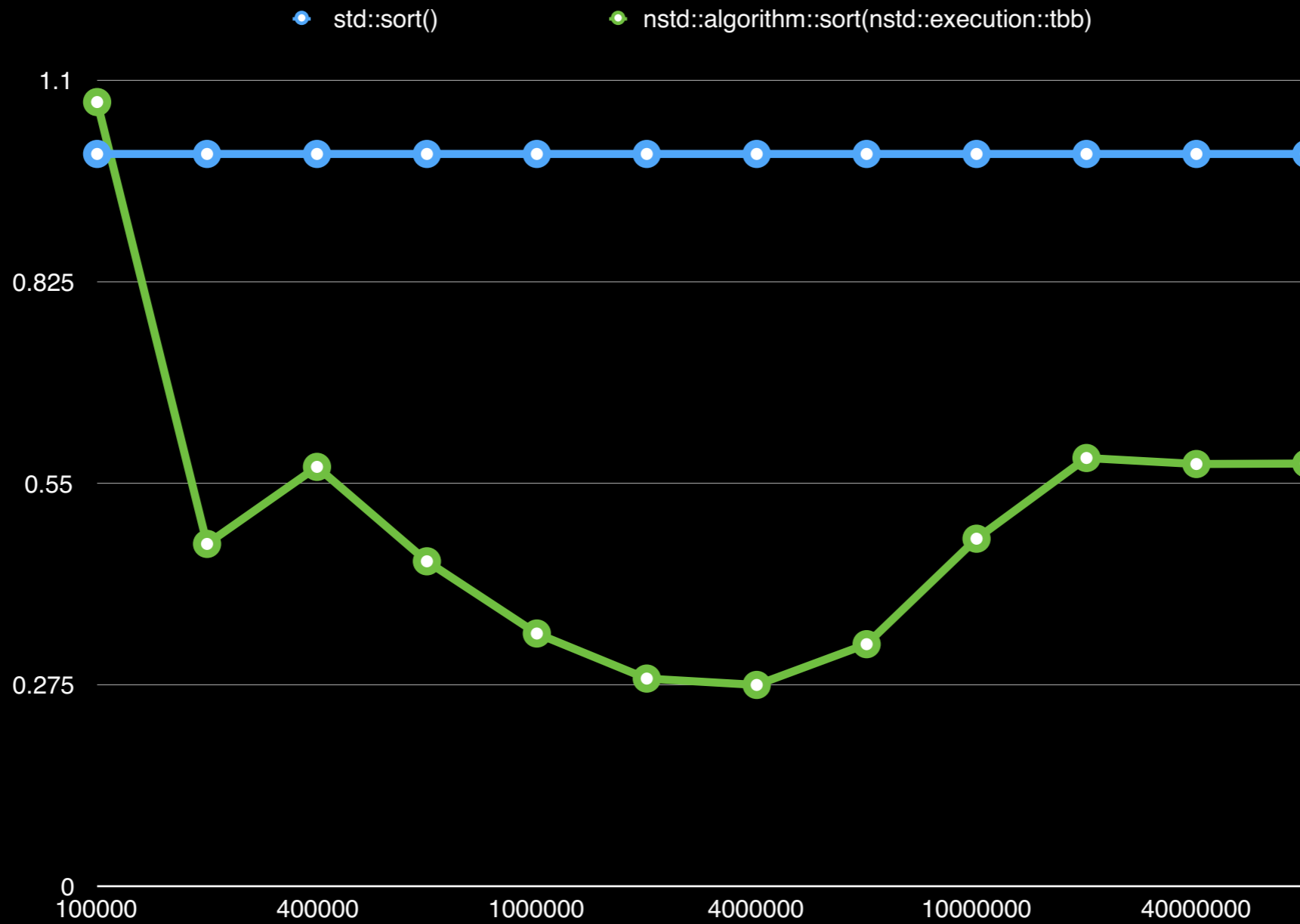
Results: sort clang 17



Results: sort gcc ARM



Results: sort clang ARM



Results: reduce

```
accumulate(begin, end);
```

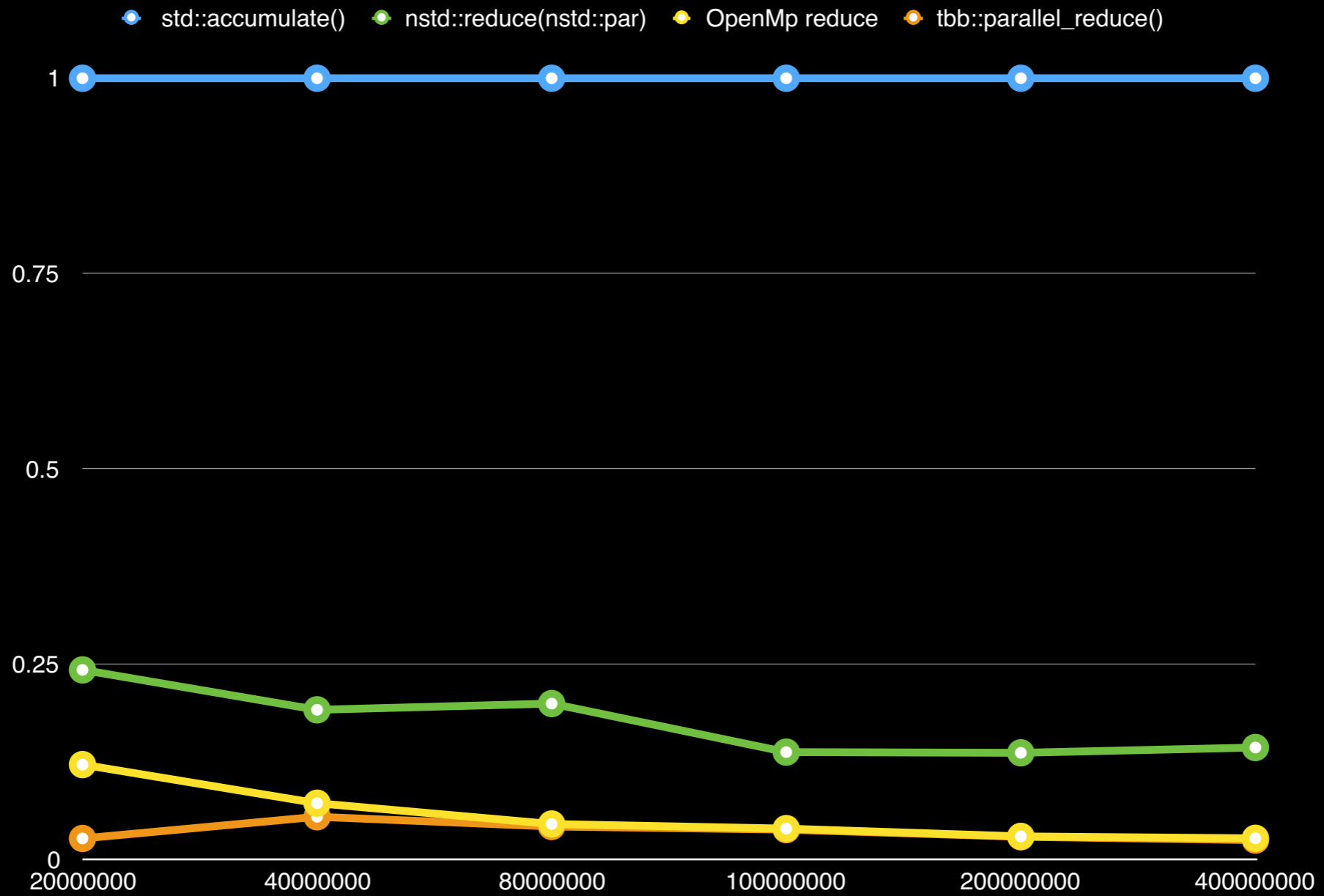
Results: reduce

```
reduce(begin, end);
```

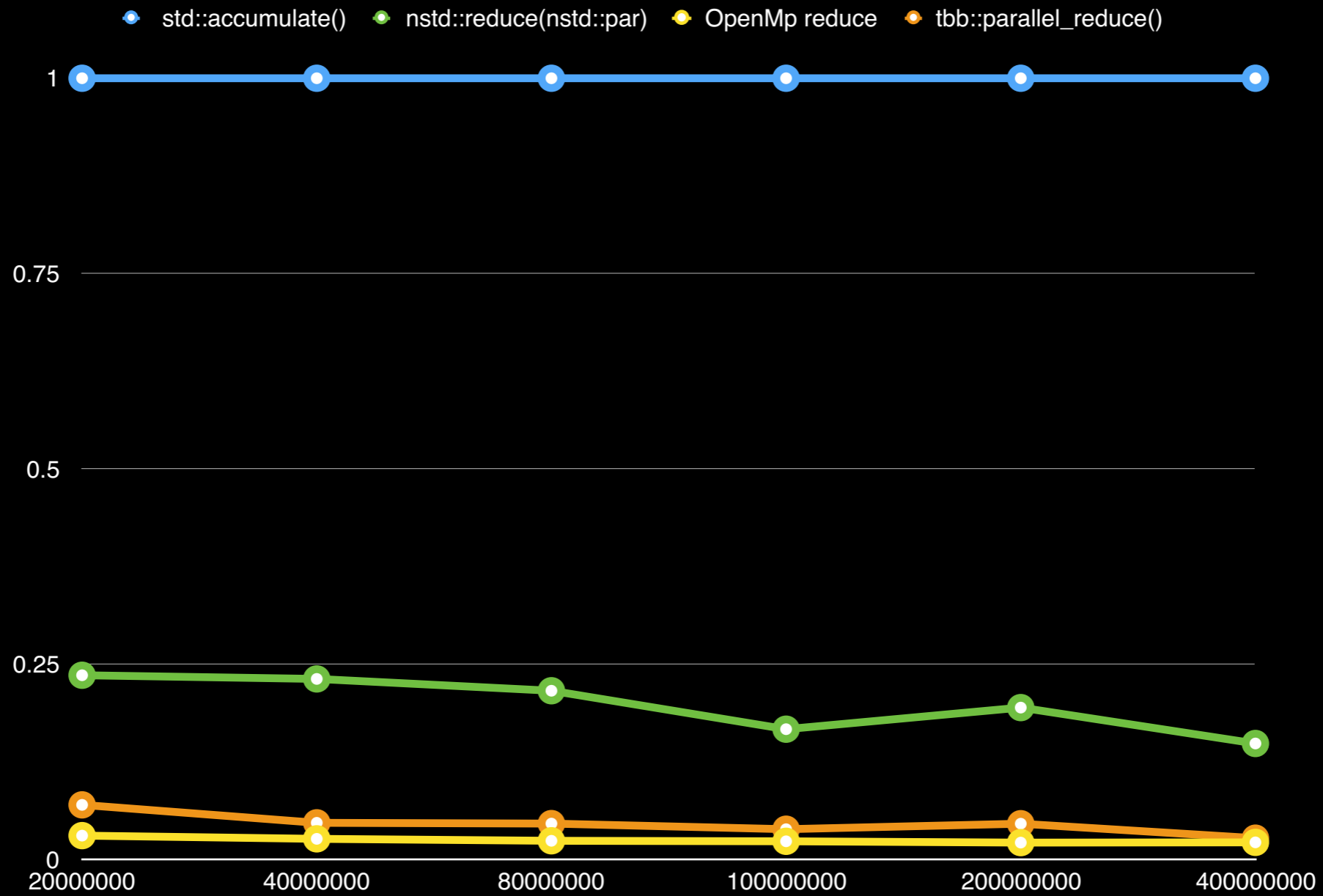
Results: redu Intel phi



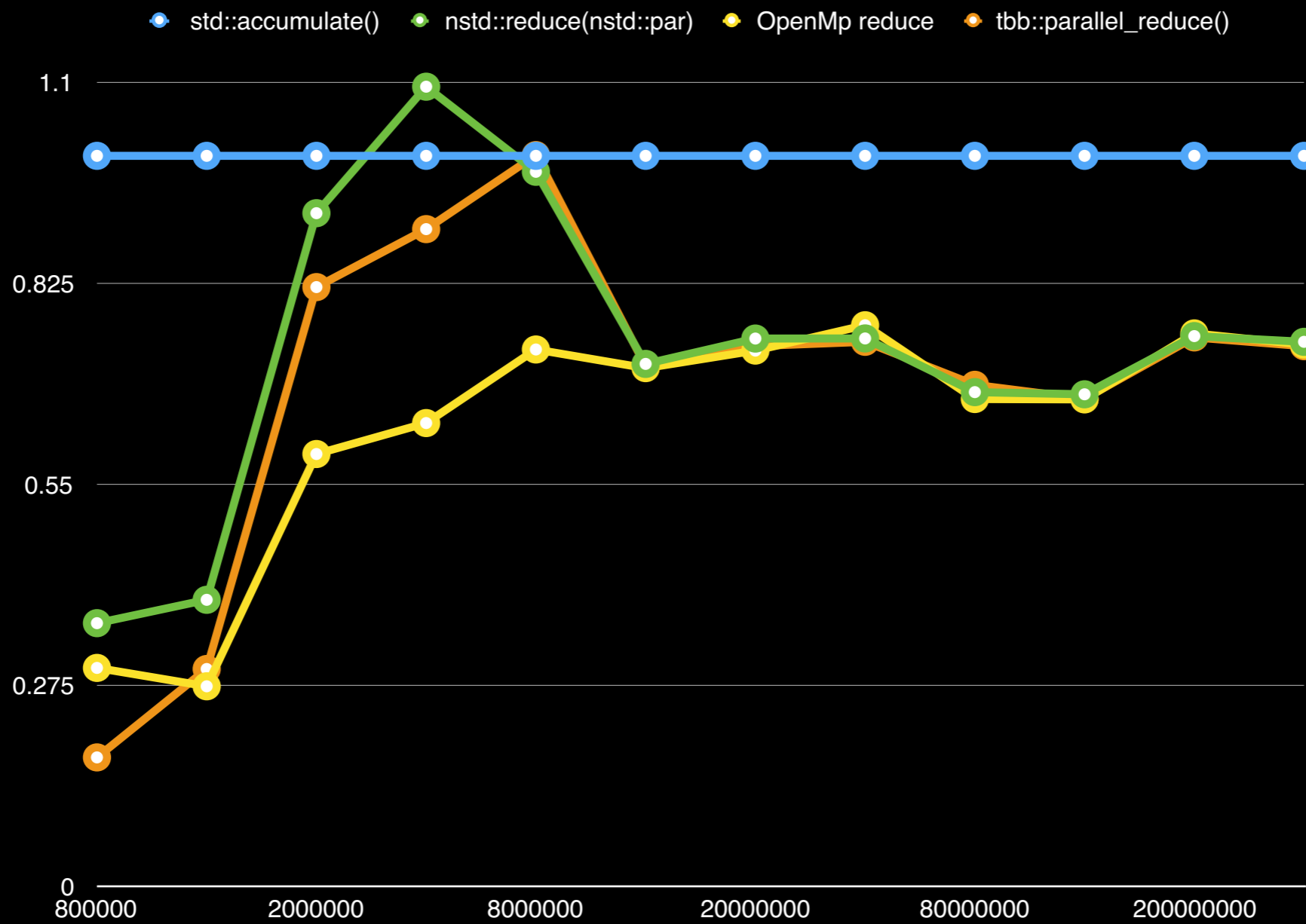
Results: redu gcc phi



Results: redu clang phi



Results: redu Intel I7



Results: redu gcc 17



Results: redu clang 17



Results: redu gcc ARM



Results: redu clang ARM



Future Directions

- more execution policies
- integration with executors
- continuation/future support
- some control over chunking

Availability

- part of C++17 standard library
- according to P0024R1 multiple implementations
 - of the parallel algorithms proposal N3554
 - all implementations seem to be partial
- not, yet, shipping with compilers

Current Implementations

- only a subset of algorithms is implemented
- typically no support for non-random access
- no support for `std::par_unseq`
- implementations don't implement a fallback

Usage Guidance

- use random access iterator if at all possible
 - for the time being the only option anyway
- it isn't worth parallelising small operations
 - sequence needs to be large
 - operations need to be expensive

Conclusions

- using STL algorithms is good
- parallel algorithms work best
 - on random access sequences
 - with large ranges
 - expensive operations

Questions

